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# Three-Dimensional Photogrammetry for Laboratory Applications

By

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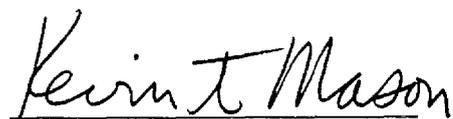
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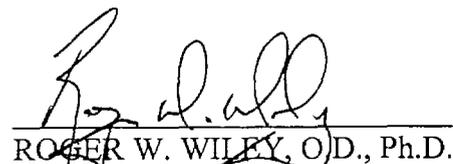
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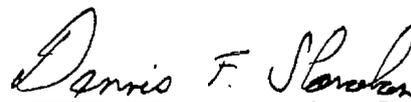


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<p>The direct linear transformation (DLT) is a method that simplifies measurements of the three-dimensional coordinates of a point target in the laboratory using photographic two-dimensional imagery. This report describes a procedure to implement the DLT equations and gives the Fortran code of computer programs for the DLT calibration of multicamera system and 3-D reconstruction of a single point from several images.</p>			
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## Introduction

Photographic coverage is the simplest method of documenting the position and motion of objects in a laboratory environment. In many cases, it is the last resort for measuring dimensions, positions, and motion during an event when other measurement methods fail to provide needed data. Photogrammetry is the use of photographic images for the measurement of lengths and coordinates of an object. For many years, three-dimensional (3-D) analytical photogrammetry has been an essential tool in surveying and map making used for accurate measurements of land features. The technique required the use of expensive metric cameras to record aerial photographs and bulky comparators to reconstruct the 3-D coordinates of points on a land surface. The analytical methods used for the reconstruction were lengthy, complex, and iterative in nature, requiring the user to provide initial guesses of the coordinates being measured. Although highly accurate, these traditional methods are not readily applicable to laboratory measurements where stereo-base photography is commonly used for recording 3-D shapes.

In the early 1970s, researchers (Abdel-Aziz and Karara, 1971) simplified the complicated analytical procedure required for the reconstruction of 3-D coordinates from planar two-dimensional (2-D) images, and introduced the direct linear transformation (DLT) between the image and object spaces. The DLT reduced the complicated calibration procedures which were required to obtain inner and outer camera parameters, and eliminated the need for expensive metric cameras, thereby opening the door for relatively inexpensive nonmetric cameras to be used in 3-D photogrammetry. Further, the DLT allowed the use of convergent stereogrammetry which is suited ideally for laboratory close-range applications.

The simplicity of the DLT method in close-range photogrammetry spawned several technologies in the fields of biomechanics, sports medicine, orthopedics, automotive safety, robotics and other fields where accurate measurements of the 3-D coordinates and motion of an object were required. In one application where the mechanical properties of aortic membrane tissue were being investigated (Melvin, Mohan, Wineman, 1975), the DLT method was used to accurately measure the coordinates on the surface of aortic membrane as it inflated (Alem, Melvin, Holstein, 1978). In another application (Schneider et al., 1985), the coordinates of landmarks on the bodies of volunteer car drivers were measured in order to describe the size and shape of an "average" driver and to develop a crash manikin which is representative of the 50th percentile male driving population.

In a recent study, researchers at the U.S. Army Aeromedical Research Laboratory (USAARL) investigated the effects of rear-surface signature of 50-caliber body armor as it defeated a round. At the same time, the viscoelastic injury criterion was being advanced (Viano and Lau, 1988) and had gained wide acceptance. Briefly, the criterion predicts an injury risk from the product  $V \cdot C$  of two measured parameters: the rate or velocity ( $V$ ) of deformation of the chest, and the amount of compression ( $C$ ) of the chest in the direction of impact. Clearly, in order to apply the viscoelastic injury criterion or evaluate the effects of the rear-surface on injury production, it would be necessary to measure the time-history of the rear surface as it deforms. This measurement then may be used to extract the speed of compression ( $V$ ) of the chest wall, and the amount of

compression (C) of the chest. Because of the configuration of impact and the speed of deformation, noncontact 3-D high speed photogrammetry is the only practical method for accurate determination of the V\*C product.

This report describes the method required to implement the DLT procedure for obtaining accurate static measurement of three-dimensional coordinates and provides a computer program which may be used for the calibration cameras pointed to a 3-D object space, and for the reconstruction of 3-D coordinates of point targets once the field has been calibrated. The method may be extended to many static and dynamic applications, including the measurement of the motion of rear-surface of the .50-caliber body armor as it defeats the penetration of a round.

### Objectives

The objectives of this study are: (1) describe the direct linear transformation for close range photogrammetry; (2) present a procedure for the calibration of a 3-D object space using the DLT equations, and provide a computer program to implement the calibration procedure; and (3) present a procedure for the reconstruction of coordinates of a point target in the calibrated 3-D space using the DLT equations, and provide a computer program which implements the reconstruction procedure.

### Methods

The direct linear transformation treats the entire chain of imaging components as a single system that reduces the 3-D object space to a 2-D image plane. Thus, we make no distinction between the camera proper and the projector. Instead, the entire camera-projector system is calibrated as a single unit. Further, because of the use of a single projector to produce images captured with different cameras, we refer to an imaging system as a "camera," with the understanding that this "camera" includes the projector as well.

For any such imaging system or "camera," it may be shown that the object space coordinates  $(x, y, z)$  of a point target are transformed to the image plane coordinates  $(u, v)$  according to the two linear equations:

$$\begin{aligned} u + xC_1 + yC_2 + zC_3 + C_4 + ux C_9 + uy C_{10} + uz C_{11} &= 0 \\ v + xC_5 + yC_6 + zC_7 + C_8 + vx C_9 + vy C_{10} + vz C_{11} &= 0 \end{aligned} \tag{1}$$

where  $(C_1, C_2, \dots, C_{11})$  are constant coefficients which depend on the imaging system being used to convert the 3-D object  $(x, y, z)$  to a 2-D image  $(u, v)$  coordinates. The imaging system is said to be calibrated when its 11 DLT coefficients are determined.

In order to calibrate a camera, i.e., compute its 11 DLT coefficients, a sufficient number of control points are placed in the field-of-view of the camera to produce at least 11 equations in the 11 unknowns. A control point is a target whose  $(x, y, z)$  coordinates are known precisely in the 3-D object space. When its image is captured and projected by a camera system, the  $(u, v)$  image coordinates of a control point also become known. Therefore, the only unknowns in the 2 linear equations are 11 DLT coefficients. Since 2 equations may be written for each control point, a minimum of 6 control points are required to produce the system of linear equations in 11 unknowns.

The only constraint on the placement of the six control points is that they must not be coplanar to ensure the existence of a solution to the system of equations. With 6 control targets, 12 equations in 11 unknowns will be written. Although one equation may be dropped before solving for the unknowns, a least square solution is preferred to minimize the error. This implies that more control targets should be used to produce an overdetermined system of equations in the 11 unknowns which may then be solved by least-squares methods. Such practice reduces potential errors in determination of  $(x, y, z)$  and the reading of  $(u, v)$  coordinates, and produces more accurate results.

Given  $n$  control point targets ( $n \geq 6$ ), we may write  $2n$  equations in 11 unknowns. The resulting system of equations is written in matrix form as:

$$\begin{bmatrix}
 x_1 & y_1 & z_1 & 1 & 0 & 0 & 0 & 0 & u_1 x_1 & u_1 y_1 & u_1 z_1 \\
 0 & 0 & 0 & 0 & x_1 & y_1 & z_1 & 1 & u_1 x_1 & u_1 y_1 & u_1 z_1 \\
 \dots & \dots \\
 \dots & \dots \\
 x_i & y_i & z_i & 1 & 0 & 0 & 0 & 0 & u_i x_i & u_i y_i & u_i z_i \\
 0 & 0 & 0 & 0 & x_i & y_i & z_i & 1 & u_i x_i & u_i y_i & u_i z_i \\
 \dots & \dots \\
 \dots & \dots \\
 x_n & y_n & z_n & 1 & 0 & 0 & 0 & 0 & u_n x_n & u_n y_n & u_n z_n \\
 0 & 0 & 0 & 0 & x_n & y_n & z_n & 1 & u_n x_n & u_n y_n & u_n z_n
 \end{bmatrix}
 \begin{Bmatrix}
 C_1 \\
 C_2 \\
 C_3 \\
 \dots \\
 \dots \\
 C_{11}
 \end{Bmatrix}
 =
 \begin{Bmatrix}
 -u_1 \\
 -v_1 \\
 \dots \\
 \dots \\
 -u_i \\
 -v_i \\
 \dots \\
 \dots \\
 -u_n \\
 -v_n
 \end{Bmatrix}
 \quad (2)$$

Equation (2) is an overdetermined system of linear algebraic equations in 11 unknowns which may be solved numerically with least squares methods. Figure 1 provides a flowchart of the procedure which may be followed for the calibration of two cameras.

Three-dimensional reconstruction simply means determination of the  $(x, y, z)$  position of a point in the calibrated 3-D object space, given the calibration constants of each camera, and the  $(u, v)$  image coordinates in each camera. This is the ultimate product of 3-D close range photogrammetry, that is, measures the coordinates of a point target in the laboratory 3-D space. To accomplish this, two or more cameras are used to capture the image of the point of interest at an

instant of time. Once the cameras have been calibrated as described earlier, the image of the same point target is digitized for each camera. This generates as many pairs of  $(u_k, v_k)$  coordinates of the same target as there are cameras. Thus, for the k-th camera, equations (1) are written as:

$$\begin{aligned} u_k + xC_{1,k} + yC_{2,k} + zC_{3,k} + C_{4,k} + u_kxC_{9,k} + u_kyC_{10,k} + u_kzC_{11,k} &= 0 \\ v_k + xC_{5,k} + yC_{6,k} + zC_{7,k} + C_{8,k} + v_kxC_{9,k} + v_kyC_{10,k} + v_kzC_{11,k} &= 0 \end{aligned} \quad (3)$$

where all terms are known except the three coordinates  $(x, y, z)$  of the target point. Therefore, a minimum of two cameras are required to produce sufficient number of equations (exactly four) which may be solved for the three unknowns. For practical reasons, several cameras are employed to cover the same field-of-view. Assuming that n cameras ( $n \geq 2$ ) are available to observe the same point, equations (3) may be written for each camera then combined into the following matrix form:

$$\begin{bmatrix} (C_{1,1} + u_1C_{9,1}) & (C_{2,1} + u_1C_{10,1}) & (C_{3,1} + u_1C_{11,1}) \\ (C_{5,1} + v_1C_{9,1}) & (C_{6,1} + v_1C_{10,1}) & (C_{7,1} + v_1C_{11,1}) \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ (C_{1,k} + u_kC_{9,k}) & (C_{2,k} + u_kC_{10,k}) & (C_{3,k} + u_kC_{11,k}) \\ (C_{5,k} + v_kC_{9,k}) & (C_{6,k} + v_kC_{10,k}) & (C_{7,k} + v_kC_{11,k}) \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ (C_{1,m} + u_mC_{9,m}) & (C_{2,m} + u_mC_{10,m}) & (C_{3,m} + u_mC_{11,m}) \\ (C_{5,m} + v_mC_{9,m}) & (C_{6,m} + v_mC_{10,m}) & (C_{7,m} + v_mC_{11,m}) \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} -(C_{4,1} + u_1) \\ -(C_{8,1} + v_1) \\ \dots \\ \dots \\ -(C_{4,k} + u_k) \\ -(C_{8,k} + v_k) \\ \dots \\ \dots \\ -(C_{4,m} + u_m) \\ -(C_{8,m} + v_m) \end{Bmatrix} \quad (4)$$

Equation (4) is an overdetermined system of  $2n$  linear equations with  $(x, y, z)$  as the unknowns, to be solved using least-squares methods. Figure 2 provides a flowchart of the procedure which may be followed for the reconstruction of 3-D coordinates from the images of two cameras.

## Results

Both calibration and reconstruction phases involve the solution of overdetermined system of linear equations using least-squares methods. The subroutine LLSQ listed in Appendix A is an implementation of such a method. It is coded for the Microsoft Fortran 5.0 compiler, but easily may be converted to other programming languages. The stability of the procedure depends on the selection of the control points and its accuracy on the number of points and the precision with which

their  $(x, y, z)$  and  $(u, v)$  coordinates were determined. A necessary constraint for the stability of the solution is for the control points to be nonplanar.

The calibration procedure outlined in Figure 1 is implemented in the subroutine DLTCAL. Appendix B provides a listing of the DLTCAL subroutine which sets up the equations and calls the LLSQ procedure to compute the 11 DLT coefficients. The subroutine is coded in free-form for the Microsoft Fortran 5.0 compiler, but easily may be modified for other languages. Input to the DLTCAL routine are the number of control points (at least six), the laboratory  $(x, y, z)$  coordinates of the control targets, and the  $(u, v)$  image coordinates of each target obtained by digitizing the image from the camera being calibrated. The subroutine returns the 11 coefficients for that camera.

The DLTCAL routine must be called for each camera being calibrated. Since all cameras must calibrate the same 3-D object space, only one array of control  $(x, y, z)$  coordinates must be used as input to the calibration routine, whereas the  $(u, v)$  array will vary from camera to camera, depending on the visibility of the target from that camera perspective.

Appendix C provides a listing of subroutine DLTREC which implements the 3-D reconstruction procedure outlined in Figure 2. The DLTREC subroutine sets up the equations and calls the LLSQ procedure to compute the three unknown coordinates. The routine is coded in free-form for the Microsoft Fortran 5.0 compiler, but easily may be modified for other languages. The reconstruction process, i.e., calls to the DLTREC routine, must be repeated for each target point. Input to DLTREC are the number of cameras (at least 2), the 11 DLT coefficients for each camera, and the image  $(u, v)$  coordinates of the target seen from each camera. The routine returns the reconstructed laboratory  $(x, y, z)$  coordinates of the target.

### Discussion

The DLT method described here is a simple and accurate method for measurement of 3-D positions. It may be applied to a single point, to several points of a surface, or across several instants of time to perform 3-D motion analysis.

In practice, several cameras are calibrated simultaneously. Each camera produces its own set of DLT coefficients which are valid only for the physical setup of the imaging system that produced the  $(u, v)$  coordinates employed in that calibration. Once the imaging system which defines a camera has been disturbed because of repositioning of the camera mount, refocusing of the camera or projector, adjustment of projection angle or scale, or change of reference system for the image plane, the 11 DLT coefficients become invalid and must be determined anew by following the calibration process. To avoid repeated calibrations, cameras usually are locked into position for the duration of a study, and are removed only when the study is completed. Alternately, a fixed-based stereo system, with two or more cameras attached to a rigid but moveable structure, may be calibrated only once and will produce 3-D coordinates relative to the attachment structure.

To improve the precision of calibration and subsequent 3-D reconstruction, and to reduce the effects of lens distortion, the  $(u, v)$  coordinates should be measured in an image reference frame whose origin is at or near the optical center of the image. Since many digitizing tablets report coordinates relative to the lower left corner of the digitizing surface, it is necessary to transform the raw digitized data using a simple origin translation. This implies the need for a repeatable procedure to ensure precise redetermination of the same image center which was used in the calibration. In the absence of fiducial marks available in metric cameras, the procedure must rely on hardware features, such as the edges or corners of the imaging gate, which are always produced during the imaging process.

Researchers (Abdel-Aziz and Karara, 1974) have proposed various models to correct imaging distortions and to account for second-order terms neglected in the linearization of the object-image transformation. However, experience has shown that additional accuracy gained by the introduction of second-order corrections is insignificant when compared to potential improvement resulting from precision and care taken in the measurements of image coordinates, and does not justify the increase in equations and solution complexities.

### Conclusions

A simple and accurate method has been provided to measure the 3-D coordinates of a point target in the laboratory using photographic coverage and the direct linear transformation. The full procedure leading to 3-D motion analysis will be presented in a separate report.

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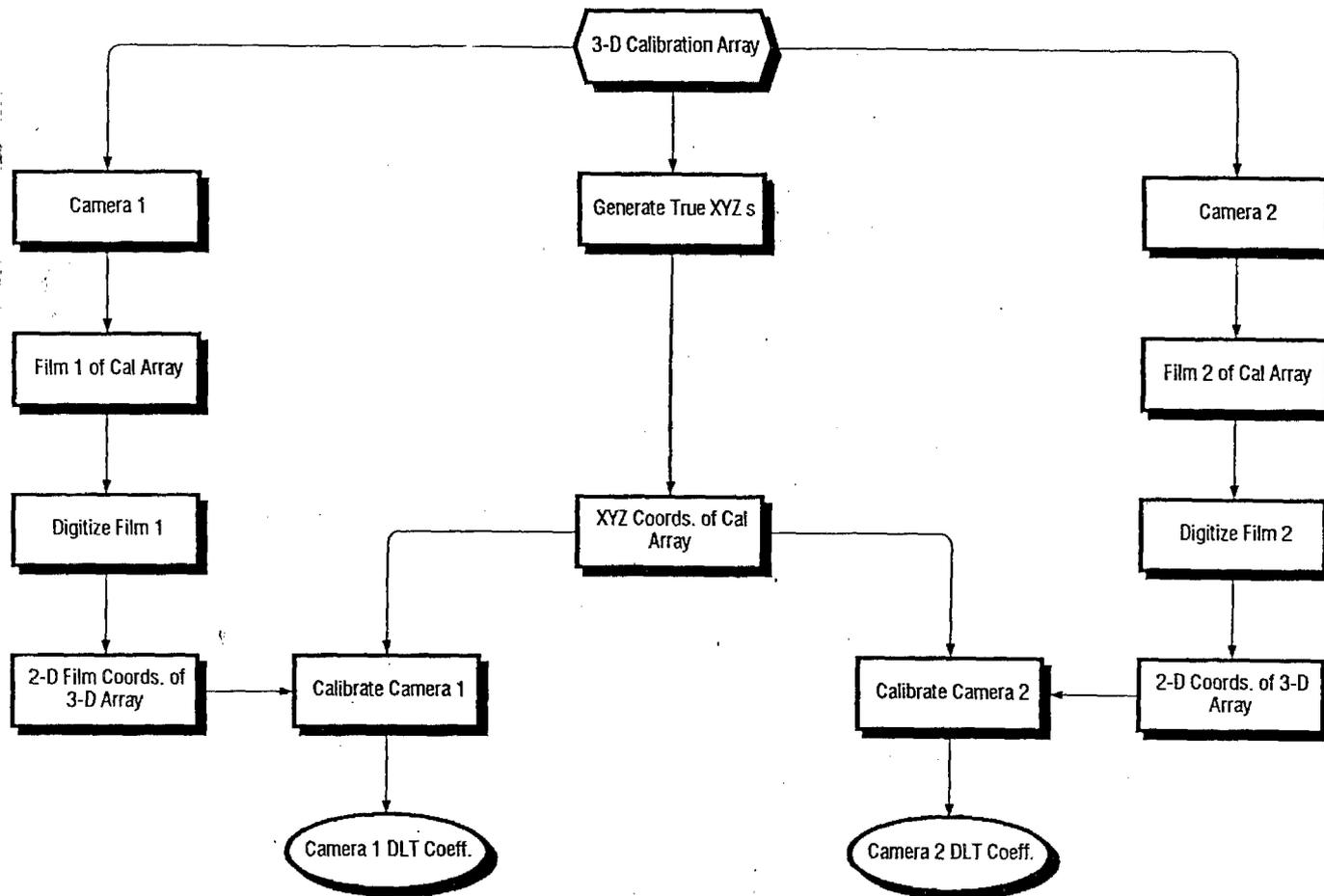


Figure 1. Flowchart of camera calibration procedure.

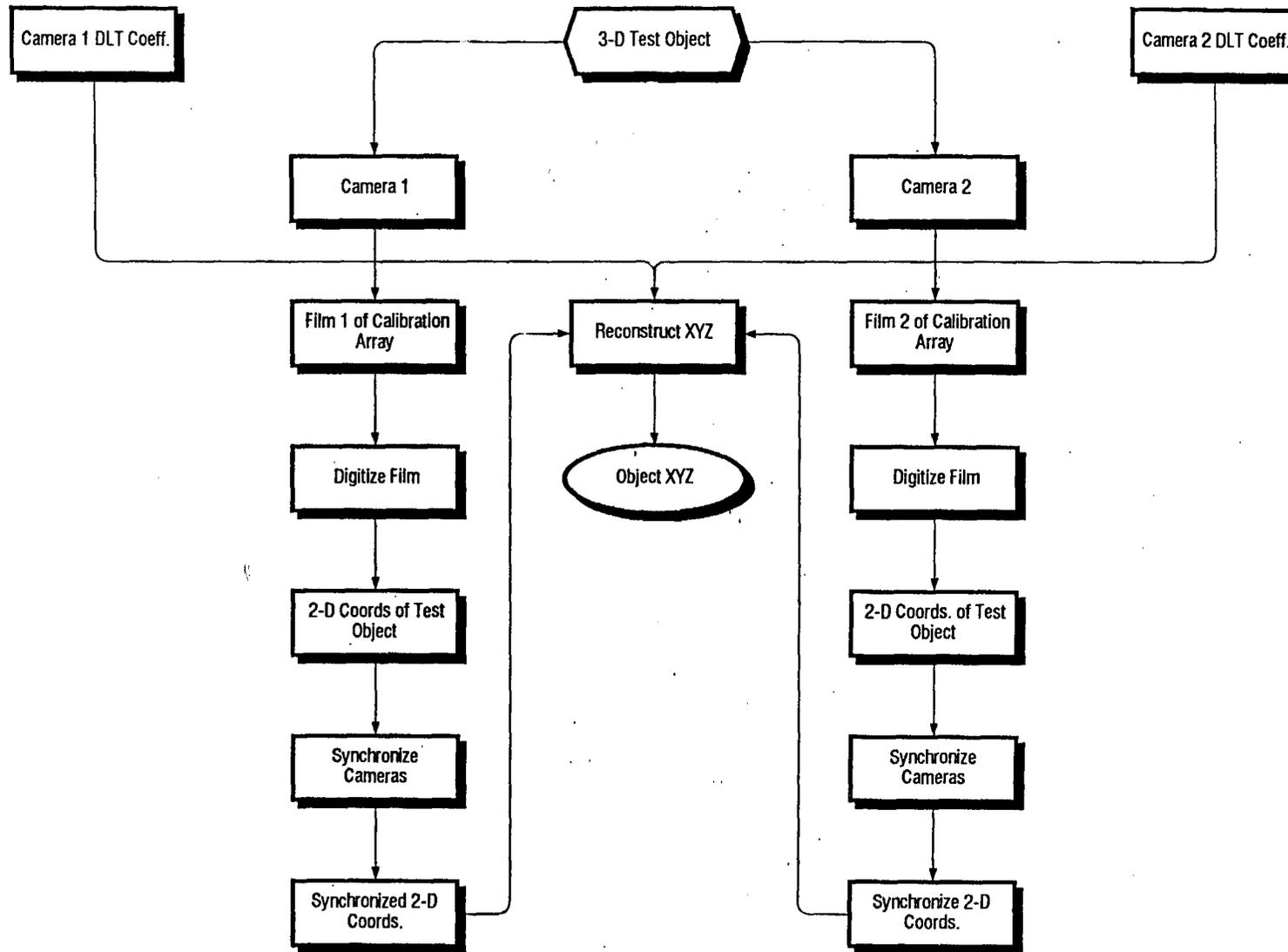


Figure 2. Flowchart of 3-D coordinates of a single point.

Appendix A.

Listing of a subroutine to solve system of linear equations.

```
*****
" Coded for MS Fortran 5.0, free-form, by N. Alem, USAARL, October 1994
"
" Purpose: To solve the system of linear equations:
"
" [A] * {X} = {B}
"
" by minimizing the euclidian norm of {B}-[A]*{X}, where [A]
" is an M by N matrix with M not less than N.
" Parameters:
" A ... M by N coefficient matrix
"
" B ... M by L right hand side matrix
"
" M ... number of rows in matrices [A] and {B}
"
" N ... number of unknown, i.e., number of columns of [A],
" which is also the number of rows in matrix {X}
"
" L ... number of columns in {X} and {B}, (usually = 1)
"
" X ... N by L solution matrix (usually N by 1 vector)
"
" IPIV ... Integer vector supplied by the use to receive
" information on column interchanges in matrix [A]
"
" EPS ... Input parameter which specifies relative tolerance
" for determination of rank of matrix [A]
"
" IER ... Error parameter. IER=0 means successful solution.
"
" AUX ... Auxiliary storage array of dimension max(2*N,L)
" On return, first L locations of AUX contain the
" resulting least squares.
"
*****

SUBROUTINE LLSQ (A, B, M, N, L, X, IPIV, EPS, IER, AUX)

DIMENSION A(*), B(*), X(*), IPIV(*), AUX(*)

IF (M .LT. N) THEN | UNDER-DETERMINED SYSTEM
  IERR = -2 | ERROR CODE
  RETURN
END IF
```

```

PIV = 0.                                ! INITIAL S(K) STORE IN AUX(K)
IEND = 0

DO K = 1, N
  IPIV (K) = K
  H = 0.
  IST = IEND + 1
  IEND = IEND + M

  DO I = IST, IEND
    H = H + A (I) * A (I)
  END DO

  AUX (K) = H
  IF (H .GT. PIV) THEN
    PIV = H
    KPIV = K
  END IF

END DO

IF (PIV .LE. 0.) THEN                    ! ZERO-MATRIX [A]
  IER = -1                                ! ERROR CODE
  RETURN
ENDIF

SIG = SQRT (PIV)                         ! TOLERANCE FOR CHECKING RANK OF [A]
TOL = SIG * ABS (EPS)

"*****" DECOMPOSITION LOOP "*****"
LM = L * M
IST = -M

DO K = 1, N
  IST = IST + M + 1
  IEND = IST + M - K
  I = KPIV - K

  IF (KPIV .GT. K) THEN                   ! EXCHANGE COLUMNS K AND KPIV
    H = AUX (K)
    AUX (K) = AUX (KPIV)
    AUX (KPIV) = H
    ID = I * M
    DO I = IST, IEND
      J = I + ID
      H = A (I)
      A (I) = A (J)
      A (J) = H
    END DO
  END IF

```

```

IF (K .GT. 1) THEN                                ! COMPUTATION OF PARAMETER SIG
  SIG = 0.
  DO I = IST, IEND
    SIG = SIG + A (I) * A (I)
  END DO
  SIG = SQRT (SIG)
  IF (SIG .LE. TOL) THEN                          ! TEST FOR SINGULARITY
    IER = K - 1                                  ! ERROR (K-1): RANK [A] LESS THAN N
    RETURN
  END IF
END IF

H = A (IST)
IF (H .LT. 0) SIG = -SIG

IPIV (KPIV) = IPIV (K)                            ! SAVE INTERCHANGE INFORMATION
IPIV (K) = KPIV
BETA = H + SIG                                    ! COMPUTE BETA AND VECTOR UK
A (IST) = BETA
BETA = 1. / (SIG * BETA)
J = N + K
AUX (J) = -SIG

IF (K .LT. N) THEN                                ! TRANSFORMATION OF MATRIX [A]
  PIV = 0.
  ID = 0
  JST = K + 1
  KPIV = JST

  DO J = JST, N
    ID = ID + M
    H = 0.
    DO I = IST, IEND
      II = I + ID
      H = H + A (I) * A (II)
    END DO
    H = BETA * H
    DO I = IST, IEND
      II = I + ID
      A(II) = A(II) - A(I) * H
    END DO
    II = IST + ID
    H = AUX(J) - A(II) * A(II)
    AUX(J) = H
    IF (H .GT. PIV) THEN
      PIV = H
      KPIV = J
    END IF
  END DO

ENDIF

```



```
IST = N + 1
IEND = 0
```

```
! COMPUTATION OF LEAST SQUARES
```

```
DO J = 1, L
  IEND = IEND + M
  H = 0.
```

```
  IF (M .GT. N) THEN
    DO I = IST, IEND
      H = H + B(I) * B(I)
    ENDDO
    IST = IST + M
  ENDIF
```

```
  AUX(J) = H
ENDDO
```

```
IER = 0
```

```
RETURN
END
```

## Appendix B.

Listing of subroutine to calibrate a camera.

```
*****
" Coded for MS Fortran 5.0, free-form, by N. Alem, USAARL, October 1994
"
" Purpose: Compute the 11 Direct Linear Transformation constants COF
" for a camera system. The COF are coefficient of the linear
" transformation which converts the three coordinates (X,Y,Z)
" of a point to its image coordinates (U,V) in that camera.
"
" Input:
" NPT ... number of control points (minimum 6)
"
" XYZ(3,*) ... array containing the 3 laboratory coordinates
" of the NPT points, known with precision.
"
" UV(2,*) ... array containing the 2 image coordinates of the
" NPT control points, digitized and referred to
" the optical center of the image.
"
" Output: COF(11) ... eleven DLT coefficients for the camera.
"
" Requires: Subroutine LLSQ to solve the overdetermined system of linear
" equations with least squares method.
"
" Note 1. An error flag is returned via the * parameter to indicate
" that the linear least squares procedure (LLSQ) failed to
" converge to a solution. In this case, the parameter EPS
" which is required inside the LLSQ but defined here should be
" increased.
"
" Note 2. An error indication also occurs if the number of points NPT
" is less than 6 or greater than 50. If more than 50 points
" are used in the calibration, the dimension of AA() array
" should be increased to (22 * NPT), and BB() to (2 * NPT)
"
" Note 3. This calibration routine must be called for each camera,
" and must be repeated if any of the cameras used in the
" system has been altered or moved.
"
*****

SUBROUTINE DLTCAL (NPT, XYZ, UV, COF, *)

REAL*4 XYZ(3,*), UV(2,*), COF(*)

DIMENSION AA(1100), BB(100), AUX(22), IPIV(11)
REAL*4 EPS /1.E-15/
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NEQ = 2 * NPT
I2 = 0

DO N = 1, NPT
  I1 = N
  I2 = I1 + 1

  AA (I1 + 0 * NEQ) = XYZ (1, N)
  AA (I1 + 1 * NEQ) = XYZ (2, N)
  AA (I1 + 2 * NEQ) = XYZ (3, N)
  AA (I1 + 3 * NEQ) = 1.0
  AA (I1 + 4 * NEQ) = 0.0
  AA (I1 + 5 * NEQ) = 0.0
  AA (I1 + 6 * NEQ) = 0.0
  AA (I1 + 7 * NEQ) = 0.0
  AA (I1 + 8 * NEQ) = UV (1, N) * XYZ (1, N)
  AA (I1 + 9 * NEQ) = UV (1, N) * XYZ (2, N)
  AA (I1 + 10 * NEQ) = UV (1, N) * XYZ (3, N)
  BB (I1) = - UV (1, N)

  AA (I2 + 0 * NEQ) = 0.0
  AA (I2 + 1 * NEQ) = 0.0
  AA (I2 + 2 * NEQ) = 0.0
  AA (I2 + 3 * NEQ) = 0.0
  AA (I2 + 4 * NEQ) = XYZ (1, N)
  AA (I2 + 5 * NEQ) = XYZ (2, N)
  AA (I2 + 6 * NEQ) = XYZ (3, N)
  AA (I2 + 7 * NEQ) = 1.0
  AA (I2 + 8 * NEQ) = UV (2, N) * XYZ (1, N)
  AA (I2 + 9 * NEQ) = UV (2, N) * XYZ (2, N)
  AA (I2 + 10 * NEQ) = UV (2, N) * XYZ (3, N)
  BB (I2) = - UV (2, N)

END DO

CALL LLSQ (AA, BB, NEQ, 11, 1, COF, IPIV, EPS, IER, AUX)

IF (IER .NE. 0) RETURN 1
RETURN
END

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IF (NCAM .LT. 2) RETURN 1
IF (NCAM .GT. 9) RETURN 1

NEQ = 2 * NCAM
I2 = 0

DO K = 1, NCAM

  I1 = K
  I2 = I1 + 1

  AA (I1 + 0 * NEQ) = COF (1, K) + UV (1, K) * COF ( 9, K)
  AA (I1 + 1 * NEQ) = COF (2, K) + UV (1, K) * COF (10, K)
  AA (I1 + 2 * NEQ) = COF (3, K) + UV (1, K) * COF (11, K)

  AA (I2 + 0 * NEQ) = COF (5, K) + UV (2, K) * COF ( 9, K)
  AA (I2 + 1 * NEQ) = COF (6, K) + UV (2, K) * COF (10, K)
  AA (I2 + 2 * NEQ) = COF (7, K) + UV (2, K) * COF (11, K)

  BB (I1) = - ( COF (4, K) - UV (1, K) )
  BB (I2) = - ( COF (4, K) - UV (2, K) )

END DO

CALL LLSQ (AA, BB, NEQ, 11, 1, XYZ, IPIV, EPS, IER, AUX)

if (IER .NE. 0) RETURN 1

RETURN
END

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