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**Design of Digital Low-pass Filters  
for Time-Domain Recursive Filtering  
of Impact Acceleration Signals**

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**January 1995**

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**United States Army Aeromedical Research Laboratory  
Fort Rucker, Alabama 36362-0577**

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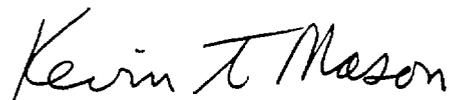
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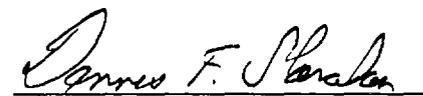


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<p>The purpose of this report is to present simplified and portable computer programs to design and implement recursive digital Butterworth filters. The design method described in the report uses the bilinear transformation to convert the well established analog design to digital formulation. Fortran codes for the design and implementation of the filter are included in the report. The resulting design is compared to another filter proposed in SAE J211 guideline for filtering impact acceleration signals. Although both design methods produced filters which are within corridors prescribed in the J211 guideline, the standard design produces a Butterworth-like response whereas the J211 design does not. Additionally, the standard design methods and the implementation algorithms allow the design of any even-order filter which may be required for filtering other long-duration signals.</p>			
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## Introduction

The U.S. Army Aeromedical Research Laboratory (USAARL) often is tasked with the assessment of injury potential from impacts and jolts. This requires the analysis of accelerations and forces obtained from transducers mounted in human-like manikins and test forms, and generated during impact tests. With the exception of the signal conditioning, the analysis is conducted at USAARL almost entirely on personal computers (PC) to perform the analog-to-digital conversion of conditioned signals, filter the digital signals, extract injury parameters from the signals, and display the results in graphical forms on the PC screen and on an attached printer.

One of the standards for processing impact signals is the Society of Automotive Engineers J211 guidelines for instrumentation for impact testing (SAE, 1994). The J211 requires signals from impact tests to be filtered using one of four channel frequency classes (CFC) of low-pass filters and specifies acceptable frequency response for each filter class. The four filters are designated as CFC 60, 180, 600, and 1000. It is clear from the J211 filter specifications that they were derived from analog Butterworth filters whose corner frequency is equal to the CFC designation divided by 0.6. The corner of a low-pass Butterworth filter is defined as the frequency at which the signal loses one-half of its power, i.e., where the signal magnitude attenuation is equal to  $\sqrt{1/2}$ , or -3 decibels (dB).

Thus, the corner of CFC 60 filter is at 100 Hz, CFC 180 at 300 Hz, CFC 600 at 1000 Hz, and that of CFC 1000 at 1650 Hz. In previous versions of the J211, acceptable roll-off slopes of filters ranged from 12 to 24 dB/octave, i.e., filters with 2, 3, or 4 poles were acceptable. The 1994 draft proposes upper and lower slopes which are 24 dB/octave, suggesting that a 4-pole filter is the basis for the requirement. The method of filtering is left up to the user and may be done with analog filters or, as is the current practice in many testing facilities, with digital filters.

Filtering is, perhaps, the most critical phase in the processing of impact signals. Its primary function is to eliminate undesired high-frequency noise that obscures the underlying signature in the signal. The importance of filtering becomes evident when considering that filtering reduces the peaks in the signal and peaks often are used for assessment of protective devices. The proliferation of personal computers has promoted the conversion of analog signals to digital ones, and increased the need for sophisticated digital signal processing algorithms to replace the functions traditionally reserved for analog electronic systems.

Because filter design formulas are well-established in the continuous-time world of electrical engineering, they often are adapted for digital filtering. Analog Butterworth filters have the property of having a maximally flat frequency response in the pass-band, and an asymptotic roll-off beyond the corner frequency. The roll-off slope is a function of the order of the filter; however, regardless of the order, the attenuation always is -3 dB at the corner frequency. More detailed description of the characteristics of these and other filters may be found in many textbooks, e.g., Oppenheim and Schaffer, 1975.

A digital filtering method which has been used at USAARL is to transform the digital signal to the frequency domain, using fast Fourier transforms (FFT), then attenuate each frequency component by an amount equal to the Butterworth function at that frequency. Since both the real and imaginary portions of the frequency magnitude are attenuated by the same amount, no phase distortion is introduced and the resulting filter is phaseless. Appendix A is a listing of a Fortran subroutine that implements this filtering method. Although this method has proven effective for most applications, it has two main disadvantages. First, because the filtering is performed in the frequency domain, there are restrictions placed by the FFT algorithm on the number of samples in the signal. Thus, the signal may not exceed a predetermined length and, often, the number of samples must be a power of 2. This means that if a longer duration signal is needed for some analysis, the sampling rate must be reduced in order to meet the limited size and longer duration requirements. Reduction of sampling rate may be tolerated up to a point below which events containing high frequencies would not be captured in the digitized signals. An example of this situation is the repeated jolts signal where several sharp impacts occur separated by time lapses that increase the overall duration of the entire signal.

Second, and a more serious disadvantage of frequency domain filtering has to do with the causality of the filter. In analog filters, the output signal is produced only as a result of an input signal. It is clear that output does not "anticipate" the oncoming step, but slowly rises as a result of it. The delayed response of this causal filter, shown in Figure 1, distorts the phase relationships between different signals and must be removed in order to synchronize the timing of events recorded in various channels. Phaseless filtering is achieved in frequency-domain FFT filtering which eliminates the time delay between input and output. Another approach to phaseless filtering in the time domain is to filter the signal once in the forward direction, then a second time in the reverse direction. It may be seen from Figure 1 that such noncausal filter unfortunately produces an output that anticipates the event and starts responding to it before it occurs. The disadvantage of this behavior is the distortion of preimpact state which is essential in some applications where the pre-impact value is used as the "zero state" of the transducer output.

Time-domain recursive filtering addresses the disadvantages FFT filtering. First, with recursive filtering, we do not have to contend with FFT algorithms that restrict the number of samples and sampling rate. The only limit is the amount of memory which may be set aside in the PC hardware. More important, filtering may be done only in the forward direction to produce causal filters. The user continues to have the option to produce a phaseless filter at the cost of losing its causality. With this flexibility, time-domain filters offer an attractive alternative to frequency-domain ones. This was recognized in the newly proposed J211 instrumentation guidelines which now include an appendix that provides the implementation of a phaseless fourth order Butterworth filter (SAE, 1994).

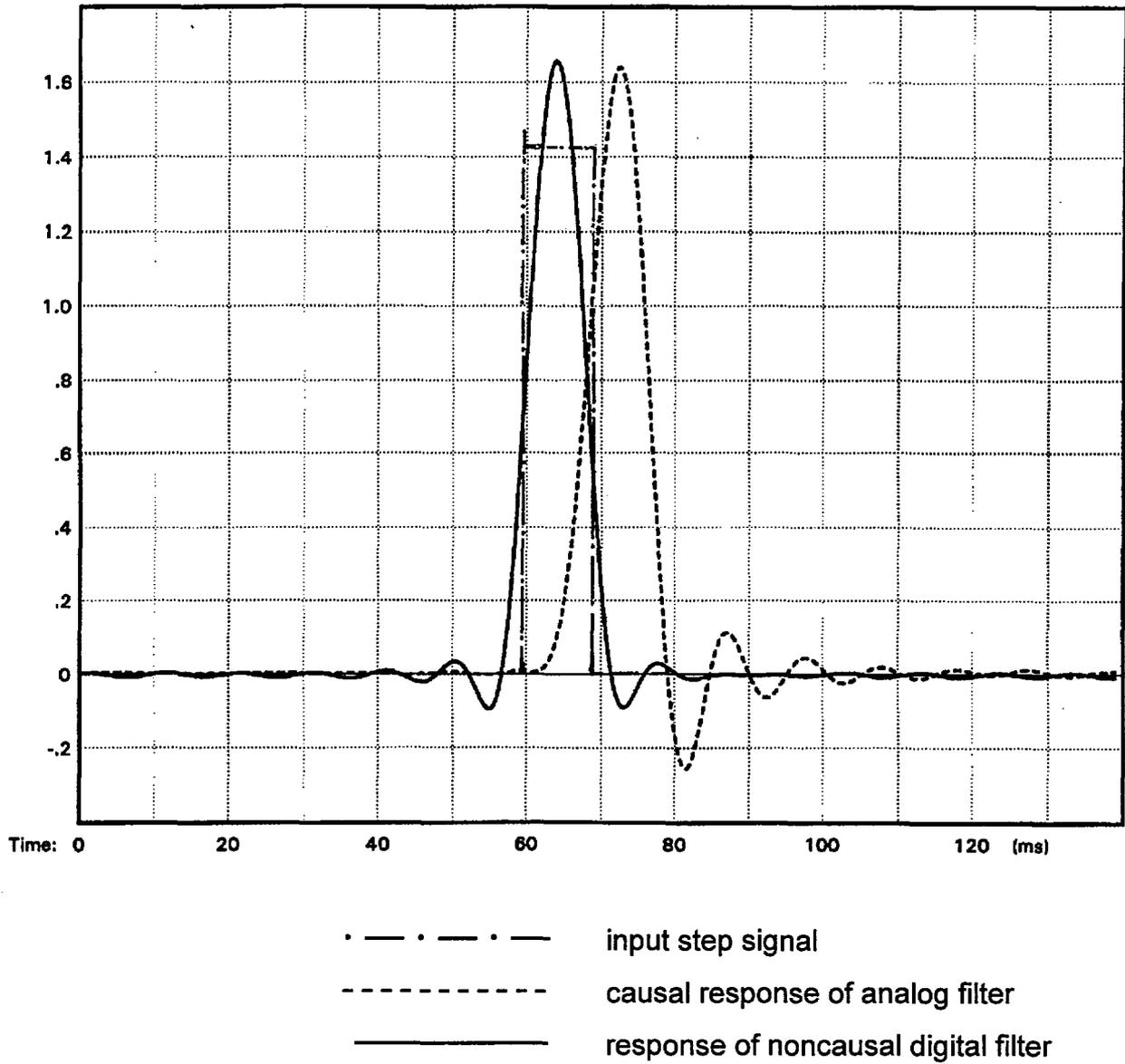


Figure 1. Comparison of causal and noncausal time responses to a step input signal.

## Objectives

- (1) To develop a computer program to design a digital Butterworth filter of arbitrary order and corner frequency for time-domain implementation.
- (2) To develop a computer program to implement the digital Butterworth filter recursively in the time domain.
- (3) To compare the frequency response of the provided design and implementation with that of the proposed J211 filters.

## Methods

The method for designing the desired digital filter is to transform the known analog filter equations into the digital domain using the bilinear transformation. Details of the procedure are described in many digital signal processing textbooks (e.g., Cappellini, Constantinides, and Emiliani, 1978) and are summarized here. The squared magnitude function of analog Butterworth filters is defined in the complex s-plane by

$$H(s)H(-s) = \frac{1}{1 + (-s^2)^N} \quad (1)$$

where N is the order of the filter and s is a complex variable. These filters have their poles in the s-plane equally spaced on a circle of radius equal to the corner (-3 dB) frequency,  $\omega_c$ .

The bilinear transformation which defined by

$$s \rightarrow k \frac{1 - z^{-1}}{1 + z^{-1}} \quad (2)$$

is a simple algebraic substitution which is applied to the Butterworth filter of equation (1). This yields the transfer function

$$H(z)H(z^{-1}) = \frac{1}{1 + \left[ - \left( k \frac{1 - z^{-1}}{1 + z^{-1}} \right)^2 \right]^N} \quad (3)$$

which can be expressed in the frequency domain by letting  $z^{-1} = e^{-j\omega}$  to give the squared magnitude of the frequency response function

$$|H(e^{j\omega})|^2 = \frac{1}{1 + \left[ k^2 \tan^2 \frac{\omega}{2} \right]^N} \quad (4)$$

For a Butterworth filter, the squared magnitude is equal to one half at the corner frequency regardless of the value of  $N$ . In particular, for  $N = 1$ , the denominator of equation (4) must be equal to 2 at the corner  $\omega = \omega_c$ , i.e.,

$$1 + k^2 \tan^2 \frac{\omega_c}{2} = 2 \quad (5)$$

This defines the constant  $k$  of the bilinear transformation for a Butterworth filter:

$$k^2 = 1 / \tan^2 \frac{\omega_c}{2} \quad (6)$$

which may substituted in equation (4) to produce

$$|H(e^{j\omega})|^2 = \frac{1}{1 + \left[ \frac{\tan(\omega/2)}{\tan(\omega_c/2)} \right]^{2N}} \quad (7)$$

Equation (7) is the squared magnitude of the digital Butterworth filter which was obtained by applying the bilinear transformation to the analog function. The real and imaginary parts of the poles of Butterworth filter, whose order  $N$  is even, are written as follows (Gold and Rader, 1969):

$$u_m = \frac{1 - x_m^2 - y_m^2}{(1 - x_m)^2 + y_m^2} \quad \text{and} \quad v_m = \frac{2 y_m}{(1 - x_m)^2 + y_m^2} \quad (8)$$

where

$$x_m = \tan \frac{\omega_c}{2} \cos \frac{2m+1}{2N} \pi \quad \text{and} \quad y_m = \tan \frac{\omega_c}{2} \sin \frac{2m+1}{2N} \pi \quad (9)$$

for  $m = 0, 1, 2, \dots, 2N-1$ . Time-domain implementation is done simply by cascading these sections to achieve the desired overall filter.

The next problem to address is the method of implementation of the filter in the time domain. In a cascade implementation, filtering will be done in stages, where output of one stage is used as input to the next one. Let  $X(n)$  be the  $n$ -th input sample in the sequence of unfiltered digital signal, and  $Y(n)$  the corresponding sample in the filtered output signal. Then, the output sample is given by the difference equation:

$$Y(n) = a_0 X(n) + a_1 X(n-1) + a_2 X(n-2) - b_1 Y(n-1) - b_2 Y(n-2) \quad (10)$$

where  $a_0, a_1, a_2, b_1, b_2$  are the coefficients of a second-order filter sections, derived from the real and imaginary parts of the poles.

The coefficients of the J211 filters are essentially those of a Butterworth design, except that the corner frequency is defined in terms of the J211 channel filter class, and an empirical factor is introduced into the equations. As with our Butterworth design, the 4th order filter is achieved by cascading two second-order sections which, in the J211 guideline, are identical. Given a signal sampled at intervals of  $T$  seconds (inverse of sampling rate in Hz), the five coefficients of a J211 filter, whose channel filter class designation is  $C$ , are given by:

$$\left. \begin{aligned} a_0 &= \frac{\omega_a^2}{(1 + \sqrt{2} \omega_a + \omega_a^2)} \\ a_1 &= 2 a_0 \\ a_2 &= a_0 \end{aligned} \right\} \begin{aligned} b_1 &= \frac{2(1-\omega_a^2)}{(1 + \sqrt{2} \omega_a + \omega_a^2)} \\ b_2 &= \frac{(1 - \sqrt{2} \omega_a + \omega_a^2)}{(1 + \sqrt{2} \omega_a + \omega_a^2)} \end{aligned} \quad (11)$$

where

$$\omega_a = \tan\left(\frac{\omega_d T}{2}\right) \quad \text{and} \quad \omega_d = 2\pi\left(\frac{C}{0.6}\right)(1.25) \quad (12)$$

The  $(C/0.6)$  is the corner frequency of the Butterworth filter and accommodates the common usage of "channel filter class" designation C instead of a corner frequency. The other 1.25 constant is an empirical constant which will be discussed later.

Finally, we will describe briefly the method used for generating the frequency response curves for this report. The magnitude response at a given frequency may be generated by passing a sine wave of that frequency through the filter and simply recording the amount of attenuation caused by the filter. By judicious selection of frequencies, a curve may be generated by connecting all the frequency response points of individual sine waves. This procedure was the basis for a computer program which was written in Microsoft Fortran to design filters, and to generate and plot frequency response curves on a personal computer. The program invokes the two filter design and implementation routines listed in Appendixes B, C, and D. Although the program allows the selection of the filtering direction (forward or backward), only forward filtering was selected. This was done after demonstrating that the direction of filtering only affects the time response of the signal, i.e., time delay of events in the signal, but not its frequency response. The curves were plotted graphically on the PC monitor display. Then a screen image capture utility was used to save the plotted response to a bitmap file. Later, a Microsoft Windows-based utility (Paint) was used to retrieve each screen image file and print it on a laser printer.

## Results

The design formulas given by equation (8) were coded into the Fortran subroutine listed in Appendix B. The recursive implementation algorithm given by equation (10) also was coded in the Fortran subroutine listed in Appendix C. The J211 filter design formulas given in equations (11) and (12) were coded in the subroutine listed in Appendix D. These three subroutines were used in a program (not included in this report) which was written to generate frequency response plots for user-specified parameters.

To test the accuracy of the design routines, four filters were designed. These are the four SAE J211 filters commonly used in processing anthropomorphic manikin transducer signals (e.g., accelerations, forces, moments ...) and obtained during impact and crash testing. The same filters were designed twice: first using the bilinear transformation (Appendix B), then using the proposed J211 formulas (Appendix D). The resulting frequency response plots are shown in Figures 2 through 5. These plots were generated under "ideal" sampling rates, i.e., such that the sampling rate of each sine wave signal was at least 10 times the frequency of the sine, or at least 10 times the corner frequency of the filter, whichever was higher.

Ordinarily, however, an analog test signal is sampled at a fixed rate which is supposed to be at least twice the highest frequency contained in the signal, but usually is 5-10 times that frequency. In order to simulate this realistic condition, a fixed sampling rate of 10,000 samples per second (10 kHz) was used to generate frequency responses of the four J211 filters using the two methods of Appendixes B and D. The 10 kHz is the sampling rate recommended in the J211 for sampling impact test signals. Results of this simulation produced the eight frequency responses are shown in Figures 6, 7, 8, and 9.

### Discussion

It is clear from the top graphs in Figures 2, 3, 4, and 5 that the standard design routine (Appendix B) and the recursive time-domain implementation algorithm (Appendix C) produce the well-known Butterworth response when the sampling rate is allowed to vary to accommodate the frequencies of the sine waves being filtered. A standard Butterworth filter has a -3 dB attenuation at the corner frequency, and an asymptotic roll-off at the rate of 12 dB/octave for each second-order section. In the standard design, the asymptote crosses the frequency axis exactly at the corner frequency, as shown in the top graphs of Figures 2, 3, 4, and 5. On the other hand, the bottom portions of the same figures demonstrate that the frequency responses produced by the J211 design formulas (Appendix D) do not result in the standard Butterworth response, even though they remain within the specified J211 response corridor.

This deviation is attributed to the 1.25 empirical constant of equation (12). Recall that J211 proposes to cascade two identical second-order sections to produce the desired fourth order filter. However, by using identical sections, the attenuation at the design corner frequency is no longer -3 dB, as expected in a Butterworth design, but doubles to -6 dB at the corner frequency. Since the frequency response is a continuously decreasing function, there exists a frequency where the fourth-order attenuation response crosses the -3 dB level. It is this frequency that the constant 1.25 tries to capture. By designing a second-order filter with a corner frequency 1.25 times higher than the desired corner of the fourth-order filter, the overall effect will be to produce an attenuation of -3 dB at the corner frequency of the desired filter. This is evident in Figures 2 through 5 (bottom graphs) where the J211 design produced the desired attenuation at the corner. Unfortunately, the intersection of the roll-off asymptote with the frequency axis does not move to the -3 dB frequency, but remains at the original corner of the second-order filter, where the attenuation is now -6 dB. In other words, the flat portion of the pass band does not extend as far as the standard design at the new corner, but starts rolling much earlier.

Although an ideal Butterworth response may be achieved when the sampling rate is unrestricted, in reality, the sampling rate is limited by hardware and software considerations to a fixed rate. For example, the J211 guideline recommends a sampling rate of 10 kHz. Using this sampling rate to illustrate its effects on the frequency response, it is clear the frequency response of the filters deviate noticeably from the ideal Butterworth response regardless of the filter design method (Figures 6, 7, 8, and 9). Since this deviation is unavoidable when the sampling rate is fixed,

it is necessary to set limits for the deviations beyond which the response would be unacceptable. The J211 corridors which are superimposed on all the figures in this report provide these limits. Of course, these limits are intended for processing force and acceleration signals from manikin crash tests, and may be redefined when other applications emerge. In addition, the deviation occurs at frequencies two to three times the corner frequency. In general, these frequencies should have been reduced already by the use of antialias analog filters prior to the analog-to-digital conversion.

Finally, the recursive filtering routine provided in Appendix C, which implements the difference equation (10), should be discussed briefly. Because two prior samples ( $n-1$  and  $n-2$ ) are required to compute each current ( $n$ ) sample, it is clear the first filtered sample that can be computed is point no. 3. Therefore, a starting method has to be devised to deal with the initial conditions of the filter. In the code provided in Appendix C, the method used is to extend the starting segment of the signal by reflecting points 2 and 3 symmetrically about point 1. This provides two additional starting points which are used to start the algorithm, then discarded. Other methods may have to be devised by the user to deal with or take advantage of specific initial conditions. Alternatively, the user may start digitizing the analog signal earlier than the event of interest, then discard the startup segment of the filtered signal in order to avoid the initial effects of the filtering process. Another comment on the method given in Appendix C has to do with memory allocation. Since one of the objectives of filtering in the time domain is to increase utilization of computer memory for long signals instead of auxiliary storage required by FFT filtering, this was accomplished in the provided code at a small cost in the algorithm complexity.

### Conclusions

A computer program was developed to design a Butterworth low-pass digital filter using the bilinear transformation. A companion program was written to implement the filter recursively in the time domain. The code for these two programs is highly portable and may be recoded in any high-level language. The two programs offer a flexible and memory efficient alternative to FFT filtering and have been demonstrated to be stable and accurate. The frequency response of the J211 filters was compared to those designed by our methods. No advantage was found in one method over the other when the sampling rate was fixed to 10 kHz. However, the ideal Butterworth filter can be achieved precisely with the bilinear transformation design program offered in this report.

### Notes on frequency response plots

- The frequency response plots included in this report were generated point by point by passing sine wave signals of different frequencies through the filter.
- For the first eight responses (Figures 2, 3, 4, and 5), the sampling rate was variable, that is each sine wave signal was sampled at a rate at least 10 times the corner frequency of the filter or at least 10 times the frequency of the sine wave being filtered, whichever was greater. For the remaining eight frequency responses (Figures 6, 7, 8, and 9), the sampling rate was fixed at 10 kHz which is the rate recommended by the J211 guideline. However, for the purpose of generating these plots, the sampling rate was never allowed to be less than 10 times the corner frequency in order to allow a sufficient number of samples per period.
- To deal with the end effects of the filter, only the middle third portions of the input and output time signals were scanned to determine the peak-to-peak span of the sine wave signals.
- The erratic behavior in some of the frequency response plots at high frequencies of both designs may be explained by the small numerical value of the peak-to-peak range in the filtered output which tends to be overcome by numerical rounding and truncation errors resulting in the observed behavior. In an actual digital signal, these high frequencies should have been attenuated already by antialias filters so that these numerical artifacts should not be significant.
- Both second-order filtering stages were done in the forward direction. Filtering direction affects the time-response, but not the frequency response of the filter.
- The straight lines above and below the frequency response curve are those defined in the J211 as boundaries of the accepted frequency response corridor. The faint straight line in the middle of the corridor is the asymptote to standard Butterworth response and has a slope of 24 dB/octave, i.e, 12 dB/octave for each of the two second-order filter sections.
- The unlabeled horizontal grid line between 0 and -10 dB corresponds to the -3 dB attenuation level.

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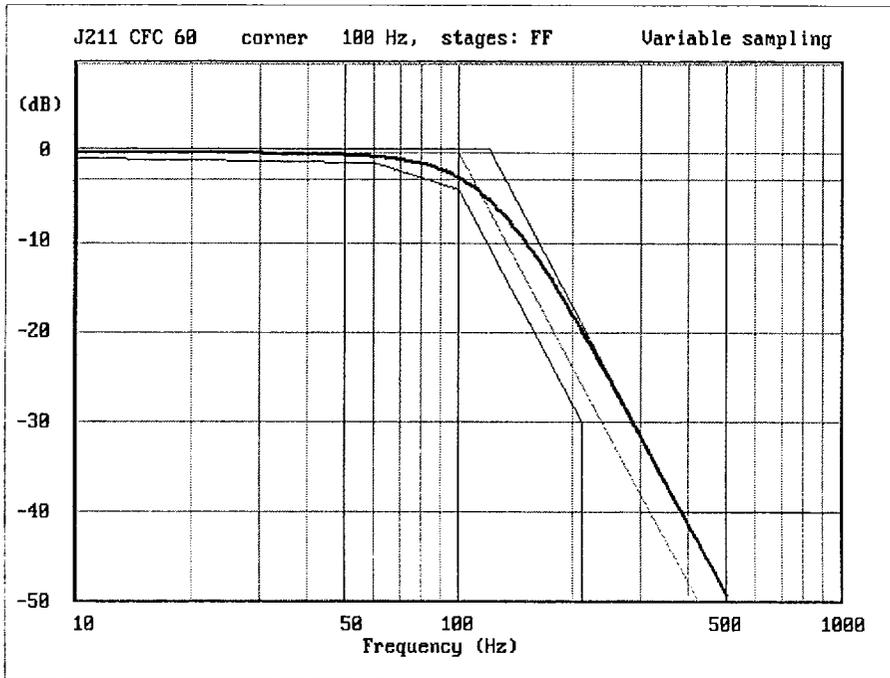
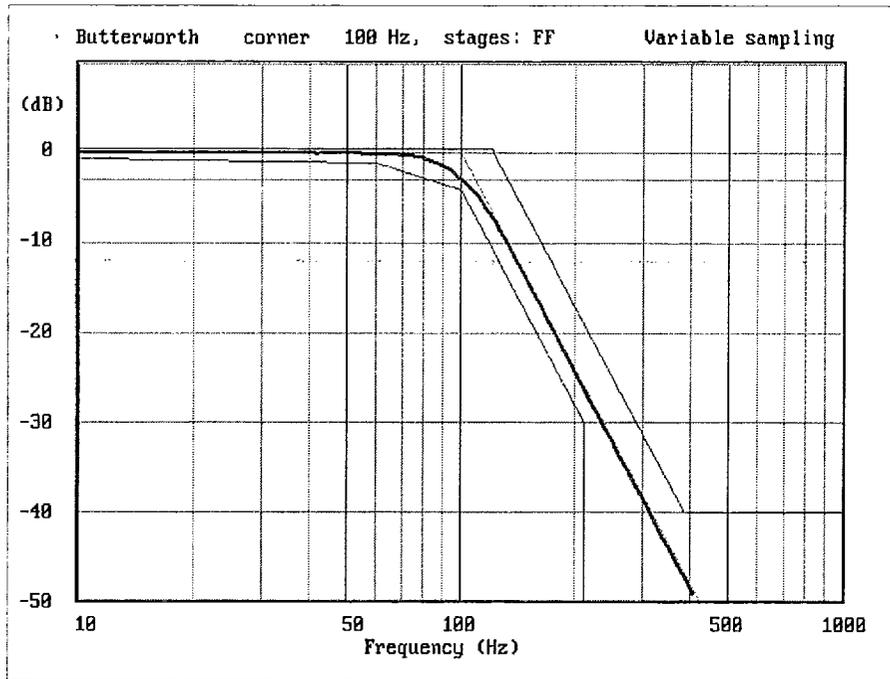


Figure 2. Frequency response of 100-Hz filter designed for variable sampling rates by standard Butterworth method (top) compared to SAE J211 CFC 60 filter (bottom).

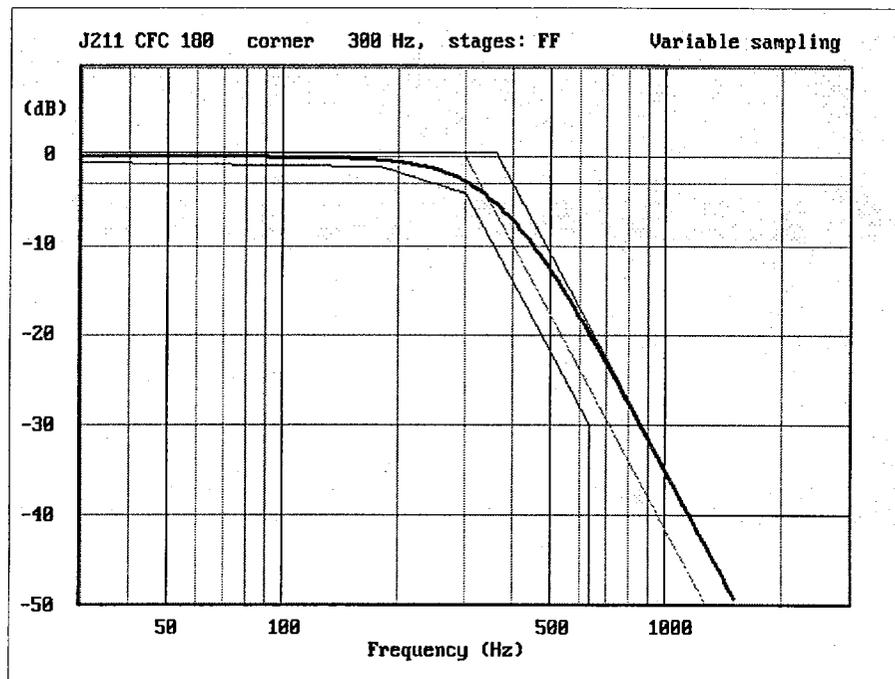
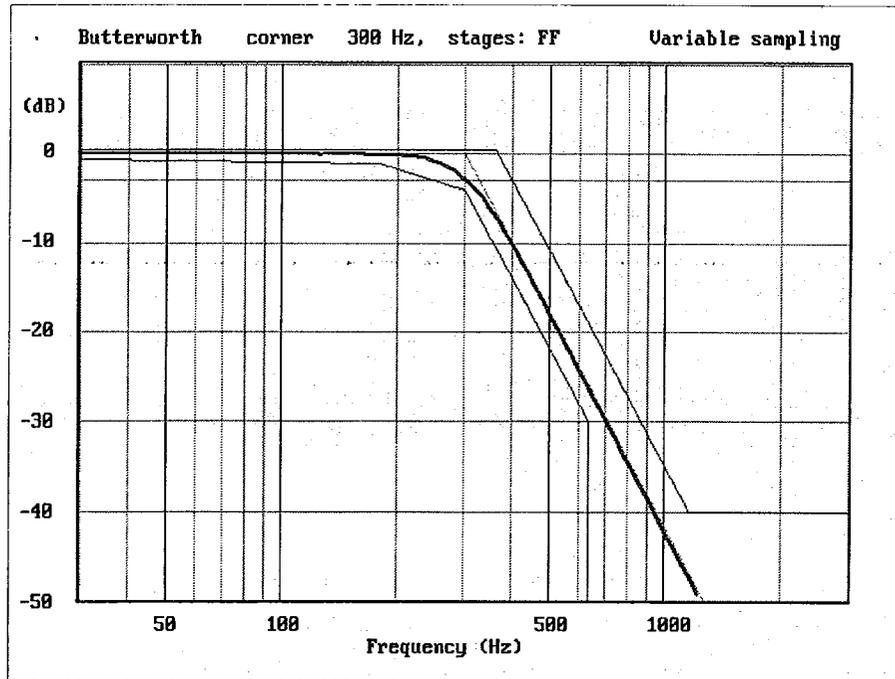


Figure 3. Frequency response of 300-Hz filter for variable sampling rates designed by standard Butterworth method (top) compared to SAE J211 CFC 180 filter (bottom).

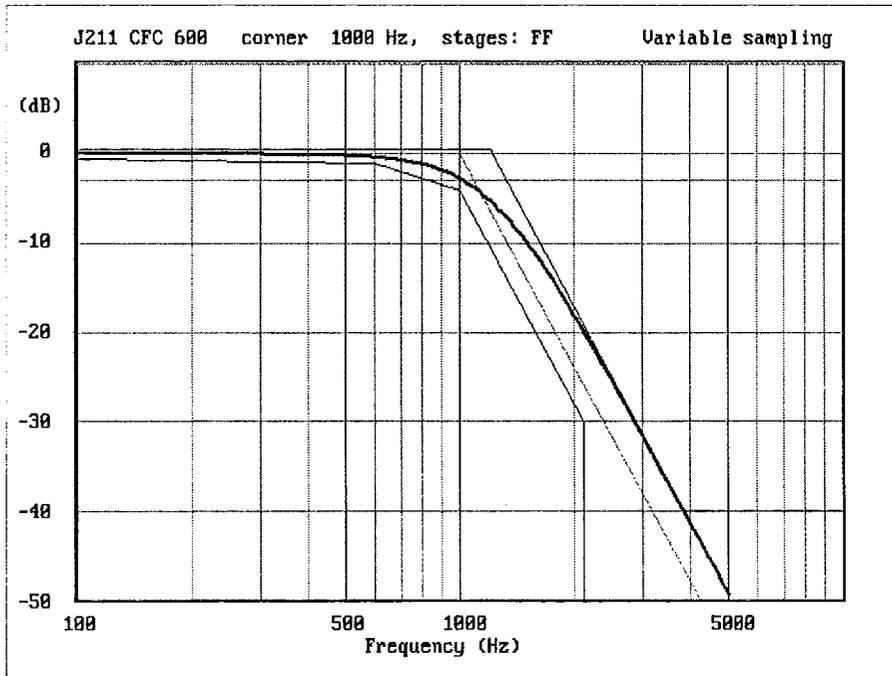
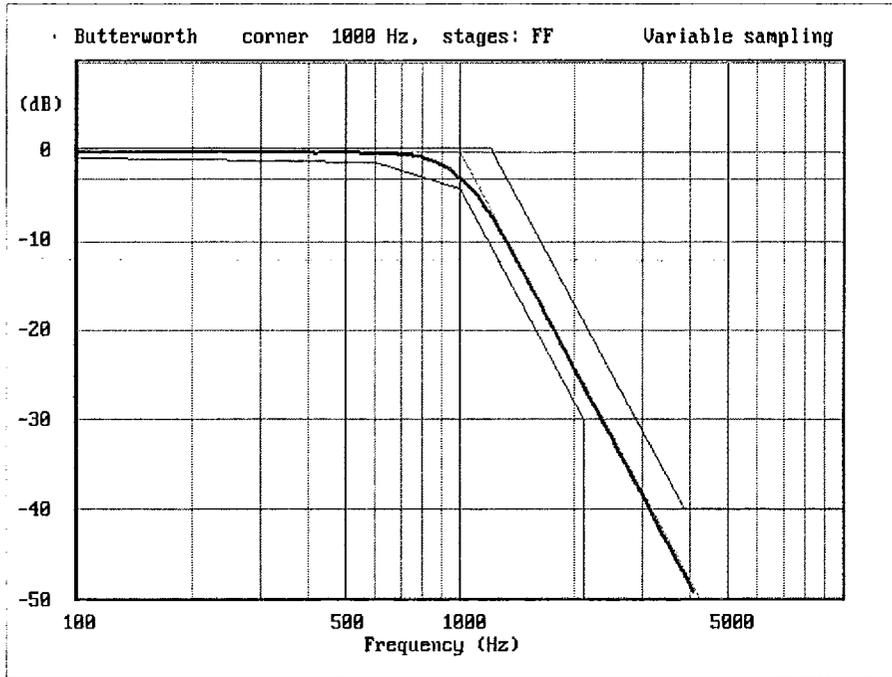


Figure 4. Frequency response of 1000-Hz filter for variable sampling rates designed by standard Butterworth method (top) compared to SAE J211 CFC 600 filter (bottom).

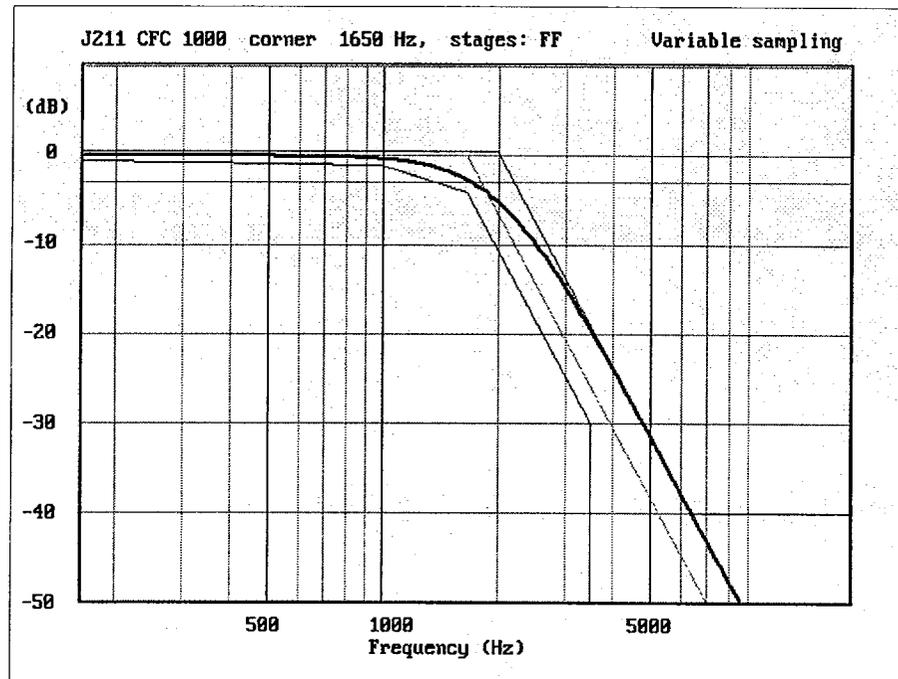
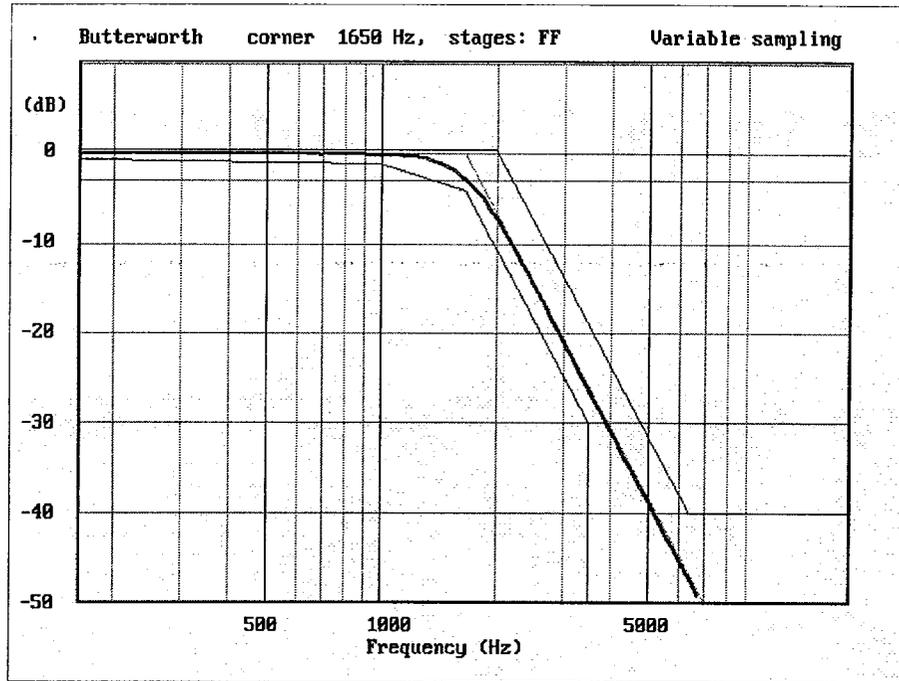


Figure 5. Frequency response of 1650-Hz filter for variable sampling rates designed by standard Butterworth method (top) compared to SAE J211 CFC 1000 filter (bottom).

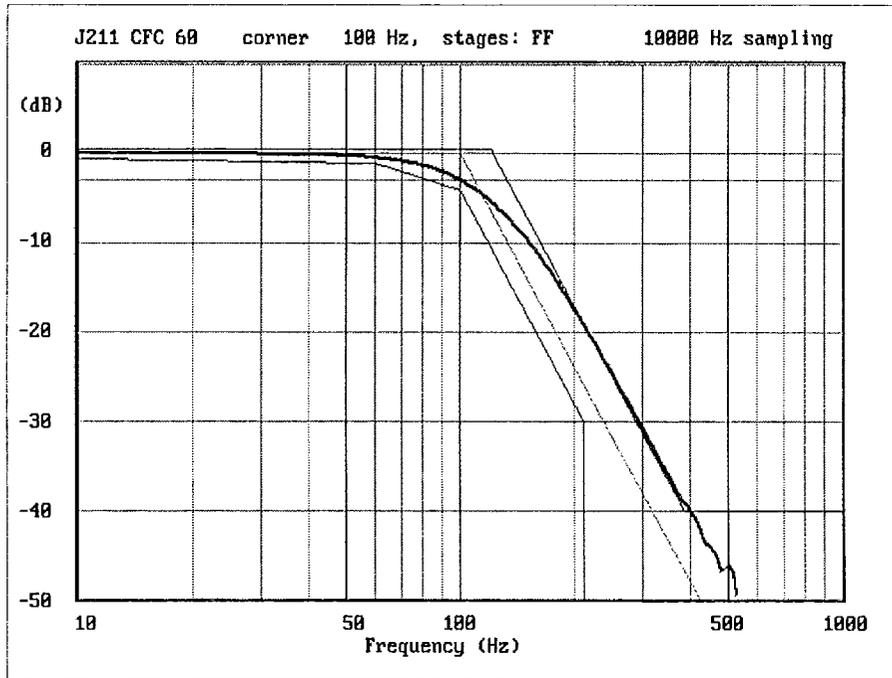
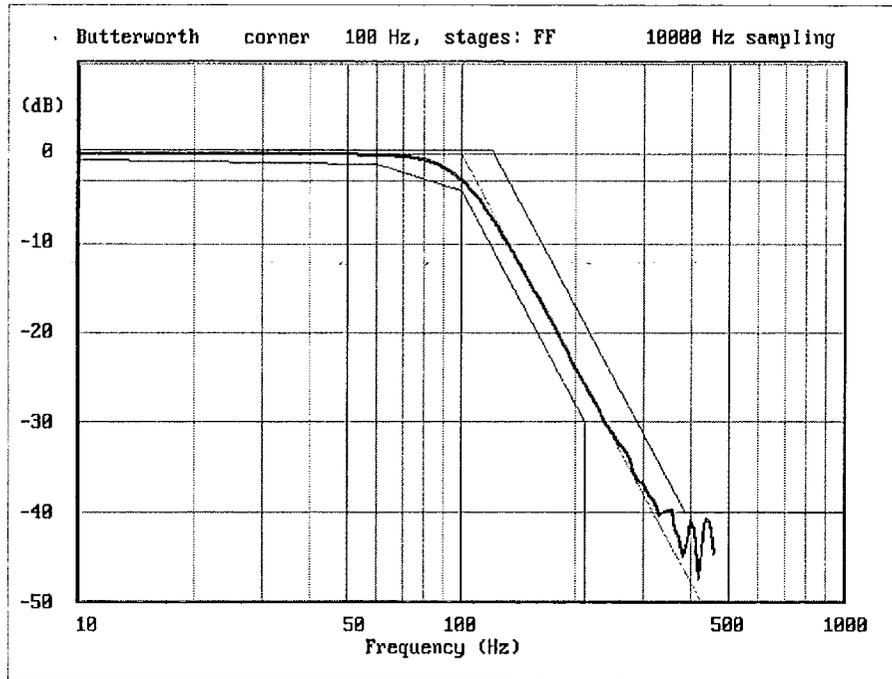


Figure 6. Frequency response of 100-Hz filter for 10 kHz fixed sampling rate designed by standard Butterworth method (top) compared to SAE J211 CFC 60 filter (bottom)

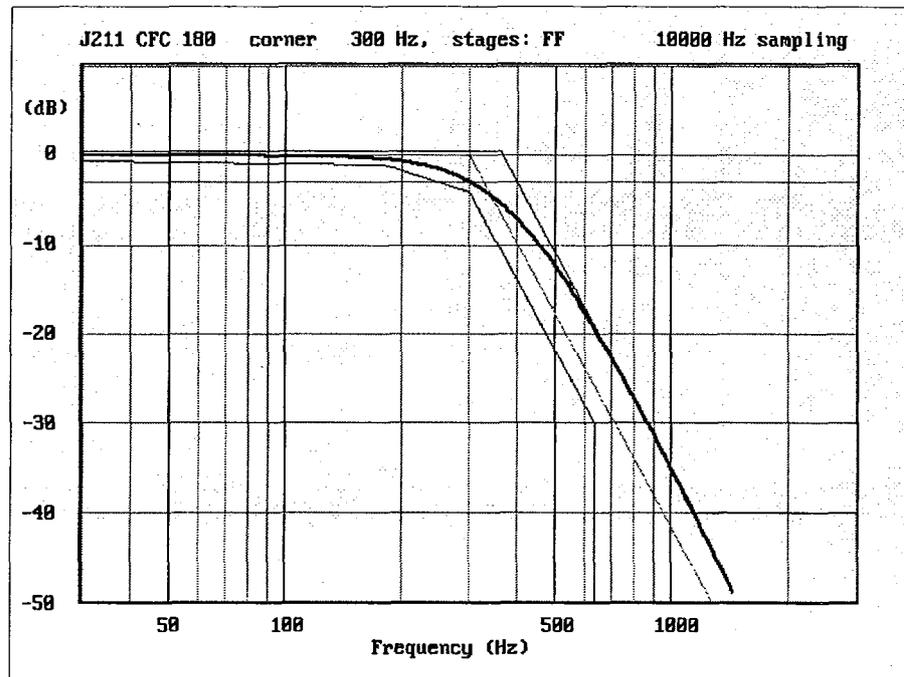
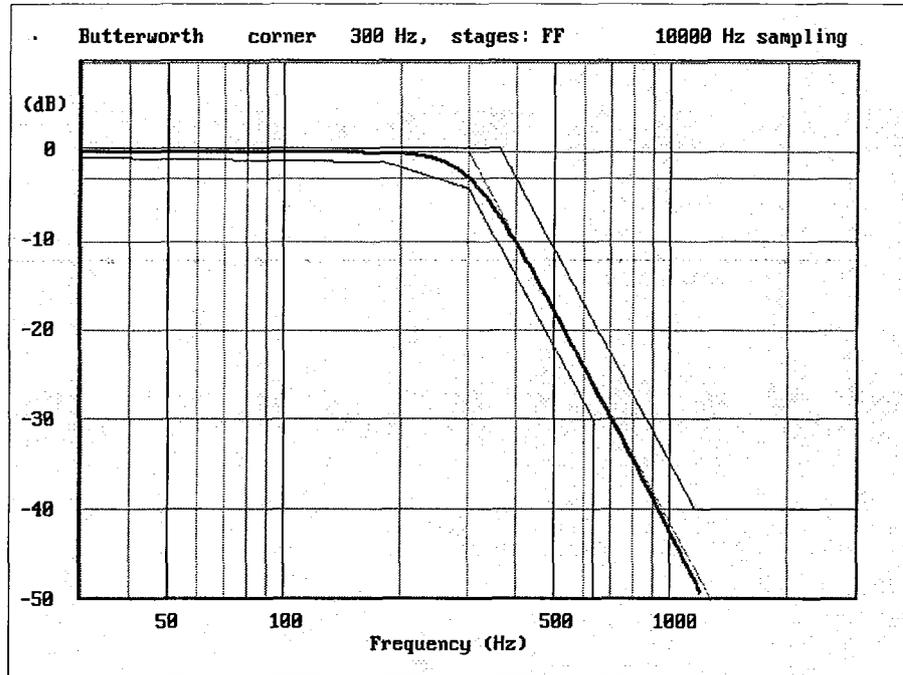


Figure 7. Frequency response of 300-Hz filter for 10 kHz fixed sampling rate designed by standard Butterworth method (top) compared to SAE J211 CFC 180 filter (bottom).

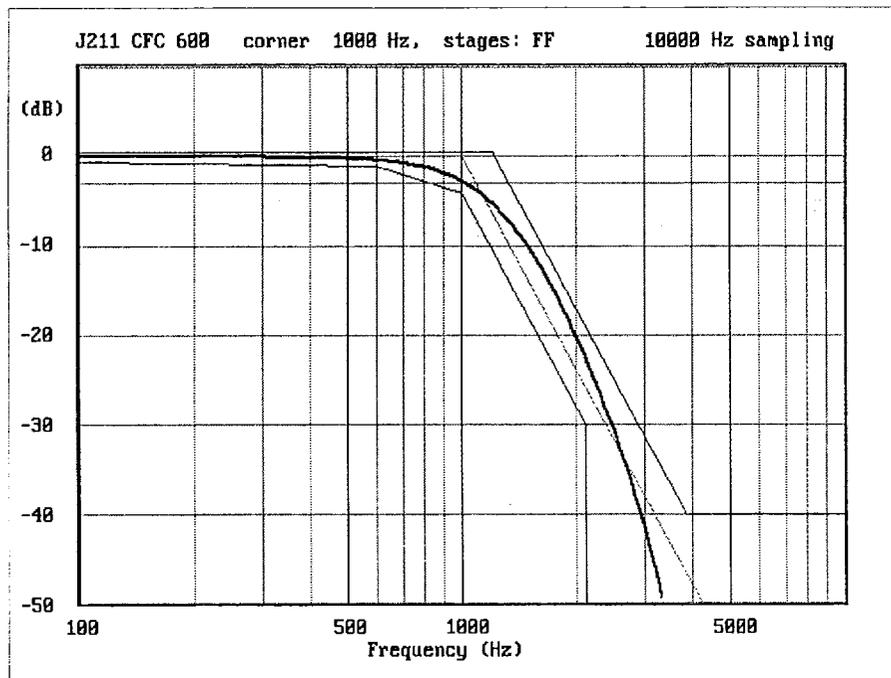
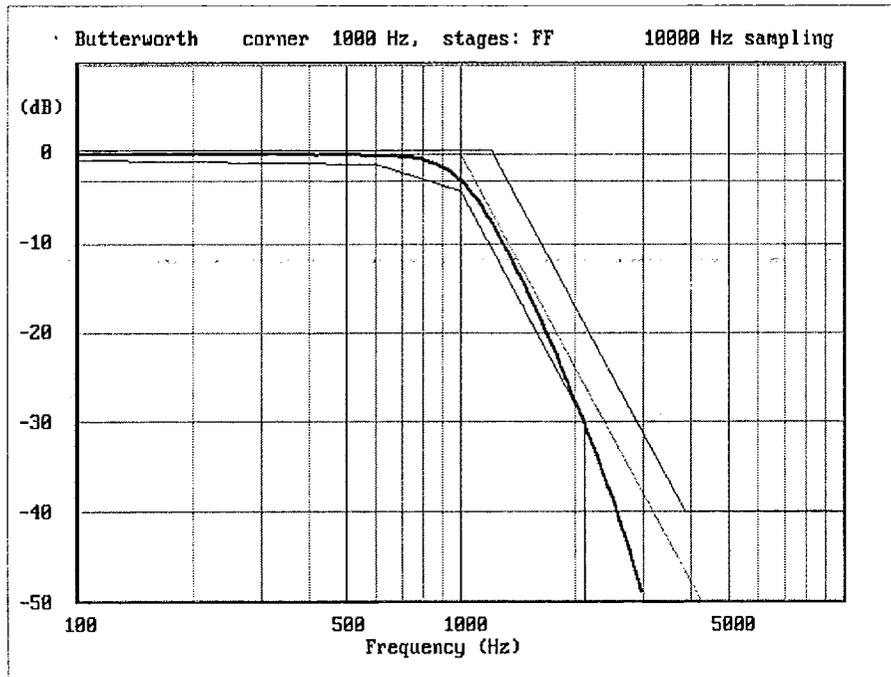


Figure 8. Frequency response of 1000-Hz filter for 10 kHz fixed sampling rate designed by standard Butterworth method (top) compared to SAE J211 CFC 600 filter (bottom).

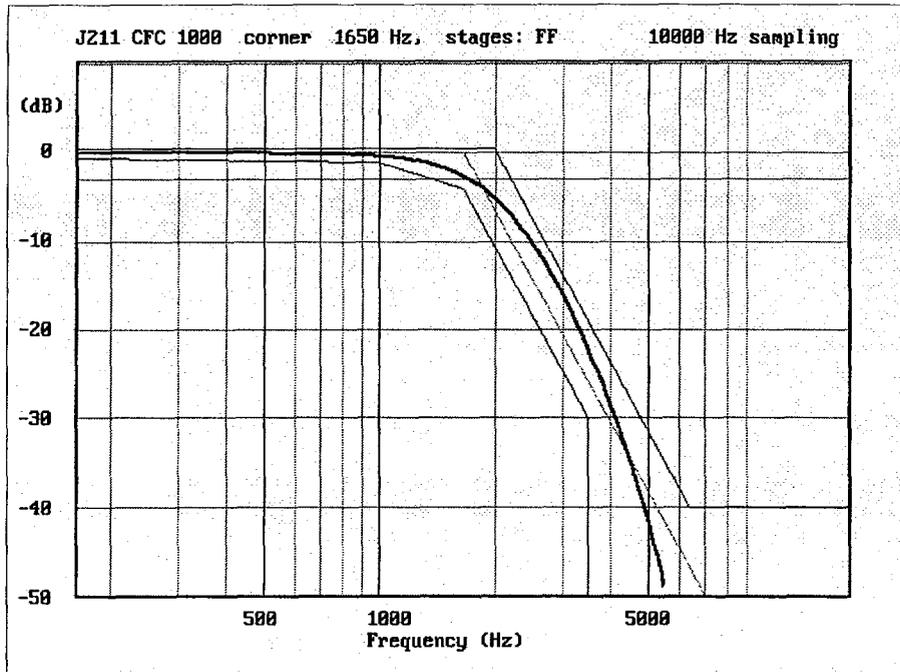
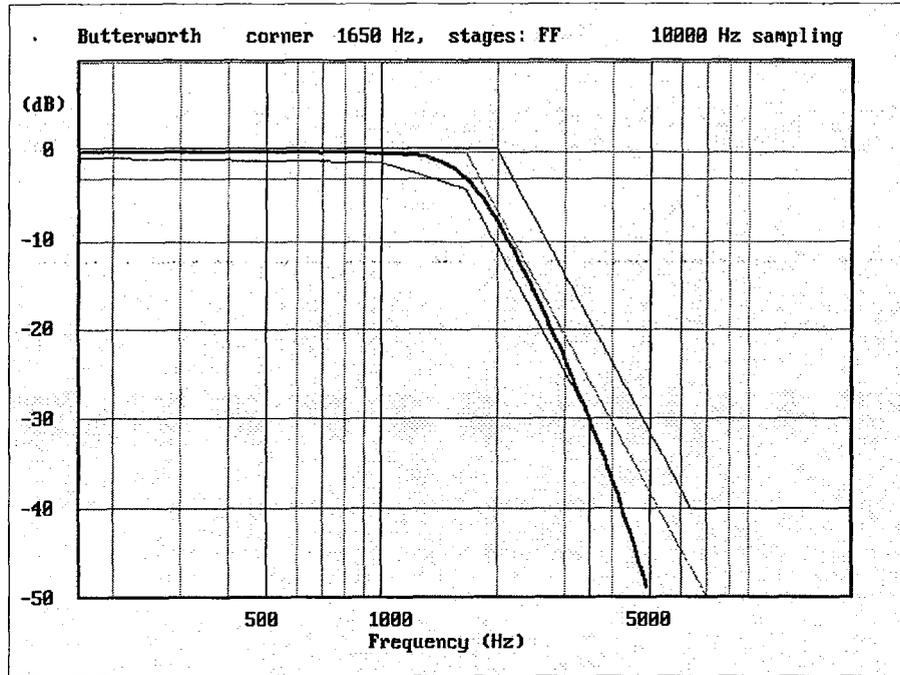


Figure 9. Frequency response of 1650-Hz filter for 10 kHz fixed sampling rate designed by standard Butterworth method (top) compared to SAE J211 CFC 1000 filter (bottom).

## Appendix A.

Program to filter in the frequency domain.

```

*****
SUBROUTINE      fft_filter (signal, npts, samhz, corner, order)
*****
"  This subroutine designs a Butterworth filter and applies it to the
"  the signal.  It requires a fast Fourier transform (FFT0 routine (not
"  provided here) because filtering is done in the frequency domain.
"
"  Parameters:
"
"    signal ... array containing signal before and after filtering
"    npts   ... number of samples in signal
"    samhz  ... sampling rate of signal, in Hertz
"    corner ... -3 dB corner of desired Butterworth filter, in Hertz
"    npoles ... number of poles of Butterorth filter, must be even
"
*****

REAL*4 signal(*)

funhz = samhz / npts
wfund = funhz / corn
order = nploes / 2
power = 2 * order
nfreq = npts / 2

CALL fft (npts, signal, +1)      ! transform to frequency domain

DO k = 1, nfreq
  ib = 2 * k
  ia = ib - 1
  wfrq = ( k-1 ) * wfund
  ginv = 1 + wfrq ** power
  gain = 1 / SQRT( ginv )
  signal( ia ) = gain * signal( ia )
  signal( ib ) = gain * signal( ib )
END DO

CALL fft (npts, signal, -1)      ! transform back to time domain

RETURN
END
```

## Appendix B.

Program to design Butterworth low-pass filters.

```

*****
SUBROUTINE      design_butter (samhz, corner, nsect, acof, bcof)
*****
"
" Subroutine to design low-pass Butterworth digital filters. The filter is
" obtained by using the bilinear transformation to transform analog filter
" equations to digital domain. Filtering is accomplished by a cascade of
" second-order sections which are defined by the order of the filter.
" Implementation in the time-domain is recursive. Arguments are:
"
" Input:
"
"   samhz ... given sampling rate (Hz) of digital signal.
"
"   corner ... given filter corner frequency (Hz) where the magnitude
"             is -3 dB (half-power point).
"
"   nsect ... given number of 2nd-order sections (pole-pairs). The
"             number of poles of the filter will be 2 x nsect.
"
" Output:
"
"   acof ... coefficients (A0,A1,A2) of 2nd-order filter sections
"   bcof ... coefficients (B0,B1,B2) of 2nd-order filter sections
"
" Implementation:
"
"   Recursive filtering through each 2nd-order section is performed by
"   the difference equation:
"
"    $Y(n) = A0 * X(n) + A1 * X(n-1) + A2 * X(n-2) - B1 * Y(n-1) - B2 * Y(n-2)$ 
"
*****

REAL*4 acof(3,*), bcof(3,*)

REAL*4 pie /3.1415926535/

wc = corner / samhz
fact = TAN( pie * wc )
npoles = 2 * nsect
sector = pie / npoles
wedge = sector / 2.
```

```

DO m = 1, nsect

  ang = wedge * ( 2*m - 1 )

  xm = - fact * COS( ang )
  ym =  fact * SIN( ang )

  den = ( 1. - xm )**2 + ym**2

  um = ( 1. - xm**2 - ym**2 ) / den
  vm = ( 2. * ym ) / den

  bcof(1,m) = 1.
  bcof(2,m) = -2. * um
  bcof(3,m) = um * um + vm * vm

  sum = bcof(1,m) + bcof(2,m) + bcof(3,m)

  acof(1,m) = sum / 4.
  acof(2,m) = sum / 2.
  acof(3,m) = sum / 4.

END DO

RETURN
END

```

Examples of filters designed with the **design butter** routine listed in this appendix

Design method: Standard Butterworth                      Filter corner = 100 Hz  
Sampling rate: samhz = 10000 Hz                              No. sections = 2

<u>Filter coefficients:</u>	<u>Section 1</u>	<u>Section 2</u>
a0 ... acof(1) =	.932544E-03	.963479E-03
a1 ... acof(2) =	.186509E-02	.192696E-02
a2 ... acof(3) =	.932544E-03	.963479E-03
b1 ... bcof(2) =	-1.88661	-1.94922
b2 ... bcof(3) =	.890340	.953070

Design method: Standard Butterworth                      Filter corner = 300 Hz  
Sampling rate: samhz = 10000 Hz                              No. sections = 2

<u>Filter coefficients:</u>	<u>Section 1</u>	<u>Section 2</u>
a0 ... acof(1) =	.754943E-02	.826380E-02
a1 ... acof(2) =	.150989E-01	.165276E-01
a2 ... acof(3) =	.754943E-02	.826380E-02
b1 ... bcof(2) =	-1.67466	-1.83313
b2 ... bcof(3) =	.704859	.866181

Design method: Standard Butterworth                      Filter corner = 1000 Hz  
Sampling rate: samhz = 10000 Hz                              No. sections = 2

<u>Filter coefficients:</u>	<u>Section 1</u>	<u>Section 2</u>
a0 ... acof(1) =	.618852E-01	.779563E-01
a1 ... acof(2) =	.123770	.155913
a2 ... acof(3) =	.618852E-01	.779563E-01
b1 ... bcof(2) =	-1.04860	-1.32091
b2 ... bcof(3) =	.296140	.632739

Design method: Standard Butterworth                      Filter corner = 1650 Hz  
Sampling rate: samhz = 16500 Hz                              No. sections = 2

<u>Filter coefficients:</u>	<u>Section 1</u>	<u>Section 2</u>
a0 ... acof(1) =	.618852E-01	.779563E-01
a1 ... acof(2) =	.123770	.155913
a2 ... acof(3) =	.618852E-01	.779563E-01
b1 ... bcof(2) =	-1.04860	-1.32091
b2 ... bcof(3) =	.296140	.632739

## Appendix C.

Program to implement a second-order filter in the time domain.

```

=====
SUBROUTINE      filter_2nd_order (x, npt, a, b)
=====
"
" Subroutine for recursive application of second-order filter to a time
" domain signal.
"
" Inside this routine, filtering is forward. Backward filtering may be
" accomplished by reversing the signal prior to calling this routine,
" then restoring the order upon return of the filtered signal.
"
" The first two points are reflected about the initial point to produce
" a reasonable starting method. Other initial conditions may dictate
" other starting methods.
"
" By sliding the filter window along the time axis, the need for auxillary
" storage is eliminated, allowing the full utilization of computer memory.
"
" This routine illustrates the correct usage of the filter coefficients
" in the difference equation:
"
"       $Y(n) = A0*X(n) + A1*X(n-1) + A2*X(n-2) - B1*Y(n-1) - B2*Y(n-2)$ 
"
" Arguments:
"
"      x()      ...  upon entry, an array containing the unfiltered signal,
"                  and replaced by the filtered signal upon return.
"
"      npt     ...  number of samples in the x() signal array.
"
"      a()     ...  array containing A0, A1, and A2 coefficients of filter
"      b()     ...  array containing B0, B1, and B2 coefficinets of filter
"                  Note: B0 must be supplied even through not used.
"
=====

REAL*4 x(*), a(*), b(*)

a0 = a(1)
a1 = a(2)
a2 = a(3)
b1 = b(2)
b2 = b(3)

```

```

xn0 = x(1)
xn1 = 2 * xn0 - x(2)
xn2 = 2 * xn0 - x(3)

yn1 = xn2
yn2 = xn1

yn0 = a0 * xn0 + a1 * xn1 + a2 * xn2 - b1 * yn1 - b2 * yn2
x(1) = yn0

xn2 = xn1
xn1 = xn0
xn0 = x(2)

yn2 = yn1
yn1 = yn0

yn0 = a0 * xn0 + a1 * xn1 + a2 * xn2 - b1 * yn1 - b2 * yn2
x(2) = yn0

yn1 = x(2)
yn2 = x(1)

DO n = 3, npt

    yn = a0 * x(n) + a1 * x(n-1) + a2 * x(n-2) - b1 * yn1 - b2 * yn2

    x(n-2) = yn2
    yn2 = yn1
    yn1 = yn

END DO

RETURN
END

```



```

ts = 1.00 / samhz
sr2 = SQRT (2.0)
wd = 2.d0 * pie * class * 2.0775

arg = wd * ts / 2.
wa = TAN (arg)
wa2 = wa * wa

den = ( 1. + sr2 * wa + wa2 )

a0 = wa2 / den
a1 = 2. * a0
a2 = a0

b0 = 1.0
b1 = 2. * ( wa2 - 1. ) / den
b2 = ( 1. - sr2 * wa + wa2 ) / den

DO m = 1, nsect

    acof(1,m) = a0
    acof(2,m) = a1
    acof(3,m) = a2

    bcof(1,m) = 1.
    bcof(2,m) = b1
    bcof(3,m) = b2

END DO

RETURN
END

```

Examples of filters designed with the design\_j211 routine listed in this appendix

Design method: SAE J211 guideline                      Filter corner = 100 Hz  
Sampling rate: samhz = 10000 Hz                      No. sections = 2

<u>Filter coefficients:</u>	<u>Section 1</u>	<u>Section 2</u>
a0 ... acof(1) =	.145237E-02	.145237E-02
a1 ... acof(2) =	.290474E-02	.290474E-02
a2 ... acof(3) =	.145237E-02	.145237E-02
b1 ... bcof(2) =	-1.88934	-1.88934
b2 ... bcof(3) =	.895153	.895153

Design method: SAE J211 guideline                      Filter corner = 300 Hz  
Sampling rate: samhz = 10000 Hz                      No. sections = 2

<u>Filter coefficients:</u>	<u>Section 1</u>	<u>Section 2</u>
a0 ... acof(1) =	.117963E-01	.117963E-01
a1 ... acof(2) =	.235925E-01	.235925E-01
a2 ... acof(3) =	.117963E-01	.117963E-01
b1 ... bcof(2) =	-1.67012	-1.67012
b2 ... bcof(3) =	.717303	.717303

Design method: SAE J211 guideline                      Filter corner = 1000 Hz  
Sampling rate: samhz = 10000 Hz                      No. sections = 2

<u>Filter coefficients:</u>	<u>Section 1</u>	<u>Section 2</u>
a0 ... acof(1) =	.971846E-01	.971846E-01
a1 ... acof(2) =	.194369	.194369
a2 ... acof(3) =	.971846E-01	.971846E-01
b1 ... bcof(2) =	-.945574	-.945574
b2 ... bcof(3) =	.334312	.334312

Design method: SAE J211 guideline                      Filter corner = 1650 Hz  
Sampling rate: samhz = 16500 Hz                      No. sections = 2

<u>Filter coefficients:</u>	<u>Section 1</u>	<u>Section 2</u>
a0 ... acof(1) =	.987937E-01	.987937E-01
a1 ... acof(2) =	.197587	.197587
a2 ... acof(3) =	.987937E-01	.987937E-01
b1 ... bcof(2) =	-.935632	-.935632
b2 ... bcof(3) =	.330807	.330807