



**An Automated Method for Determining
Mass Properties**

By

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Introduction

Mass property evaluations are an integral part of the engineering analysis performed by the Biodynamics Research Division at the U.S. Army Aeromedical Research Laboratory (USAARL). For man-mounted equipment, it is important to define and measure the mass properties in order to accurately develop math models, understand human performance impacts, and to conduct comparative evaluations. To measure these properties, the Space Electronics KSR330-60 Mass Properties Instrument (MPI) was selected. The purpose of this paper is to describe the MPI's operating theory and the procedure to measure mass properties with the MPI.

Equipment

The KSR330-60 MPI was designed and produced by Space Electronics, Inc. The MPI is capable of testing a 330-lb load. The maximum full-scale moment allowed due to the offset of the part's CM is 60 in-lb. The accuracies of the MPI are listed in Appendix A. The MPI requires a continuous source of pressurized (70 to 95 PSI) nitrogen or dry clean instrument air. A JUN-AIR 2000 oil-less air compressor with an absorption dryer was selected to supply instrument air to the MPI. The MPI operation is fully automated with a personal computer and a dot matrix printer. The MPI system setup is shown in Figure 1.

Theory

The MPI is designed to obtain accurate measurements of the center-of-mass (CM) and the mass moment of inertia (MOI). The CM is defined as the point in a body at which its entire mass may be assumed to be concentrated. The MOI of a body is defined as a measure of its resistance to rotational acceleration.

Center-of-mass

For each orthogonal axis of a test part, its CM is calculated by measuring the force required to balance the test part on the test platform's pivot axis. This pivot axis serves as the fulcrum for the test platform, which is suspended by a rotary gas bearing. The test part's mass and the unknown distance between the test part's CM location and the test platform's pivot axis results in a moment applied about the pivot axis (Figure 2). The MPI measures the force required to balance

* See manufacturer's list.

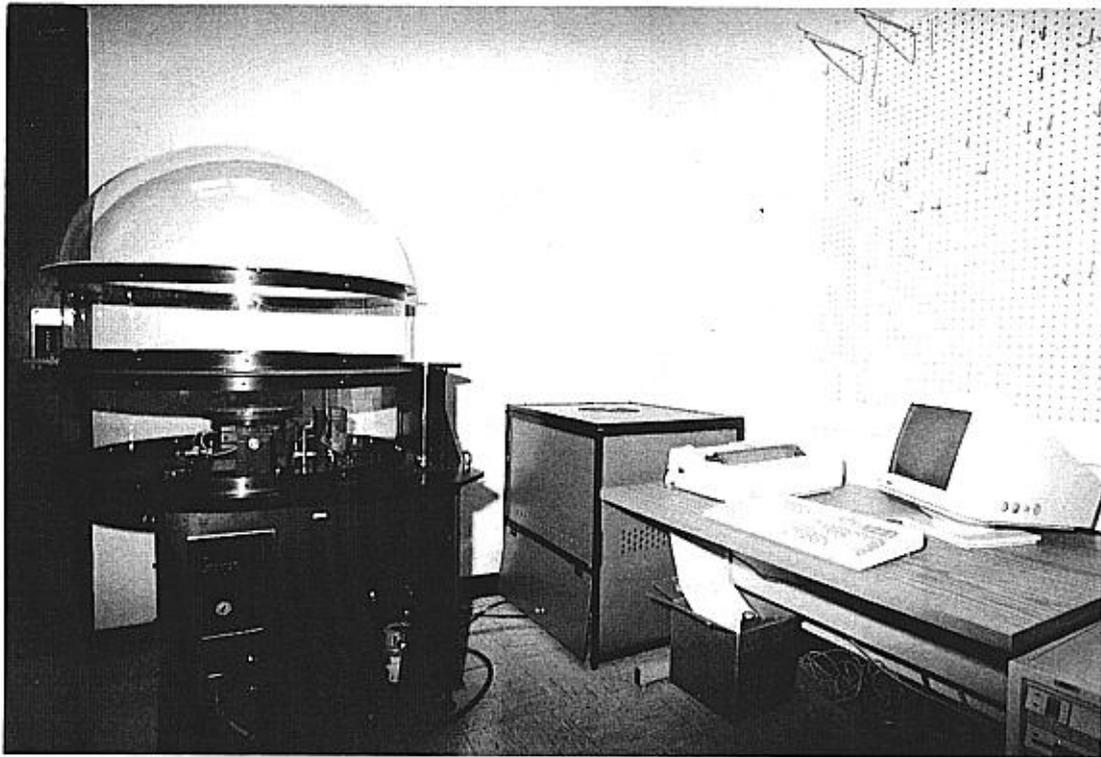


Figure 1. KSR330-60 Mass Properties Instrument setup.

this moment caused by the test part. This balancing force (F_1) is sensed by a force transducer located at a known distance (d_1) from the fulcrum. From the moment equation,

$$M_1 = F_1 d_1,$$

the moment (M_1) produced by the transducer relative to the pivot axis can be determined. In order to balance the test platform, this moment must equal the moment resulting from the CM of the test part. Since the force applied by the test part (F_2) is known from its mass, the unknown distance from the fulcrum to the CM of the test part (d_2) is easily calculated from the following equation:

$$F_1 d_1 = F_2 d_2$$

$$\text{or } d_2 = (F_1 d_1) / F_2.$$

Moments of inertia

Mass moments of inertia

The MOI calculations are based on the period of rotational oscillation of the test platform, which is configured as an

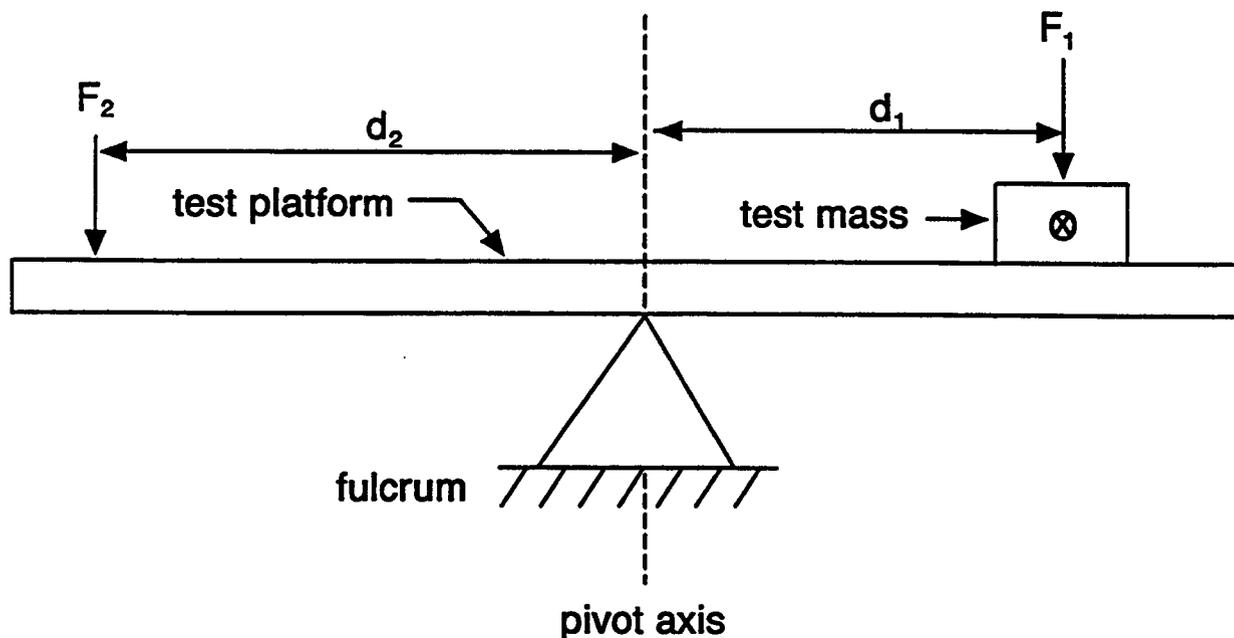


Figure 2. Moments produced during CM testing.

inverted torsional pendulum (Figure 3). To begin the oscillation, the platform is rotated slightly and released. The test platform's MOI is related directly to the oscillation period (T) by the following equation:

$$I = CT^2,$$

where C is a physical constant dependent on the torsional pendulum configuration. This constant is found experimentally as shown in Appendix B.

To determine the MOI of the test part, a tare period (T_t) must first be measured for the platform and any test fixture necessary to support the test part. The test part then is added to the system and another period (T_i) is acquired. Using the calibration constant, C, the test part MOI then is calculated:

$$I = C(T_i^2 - T_t^2).$$

This measurement and calculation provide the part's MOI about a single axis. This procedure must be repeated for the part's two remaining axes. These moments, I_x , I_y , and I_z , are used to determine the part's principal moments of inertia and principal axes of inertia. The MOIs are mathematically represented by:

$$I_{xx} = \int (y^2 + z^2) dm,$$

$$I_{yy} = \int (z^2 + x^2) dm,$$

$$I_{zz} = \int (x^2 + y^2) dm.$$

where x , y , and z are the distances from the basic coordinate axes of the CM to the differential mass (dm) (Figure 4).

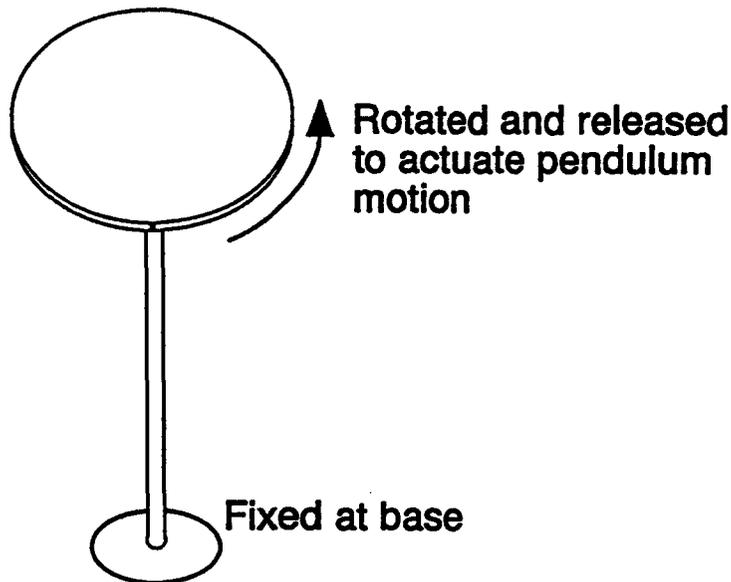


Figure 3. Inverted torsional pendulum.

Mass products of inertia

Along with the MOIs, the products of inertia are required in order to calculate the principal moments of inertia and the principal axes of inertia. The products of inertia, P_{xy} , P_{xz} , and P_{yz} , are mathematically defined by the following integrals:

$$P_{xy} = P_{yx} = \int xy dm$$

$$P_{xz} = P_{zx} = \int xz dm$$

$$P_{yz} = P_{zy} = \int yz dm$$

where x , y , and z are the distances from the basic coordinate axes of the CM to the differential mass (dm) (Figure 4).

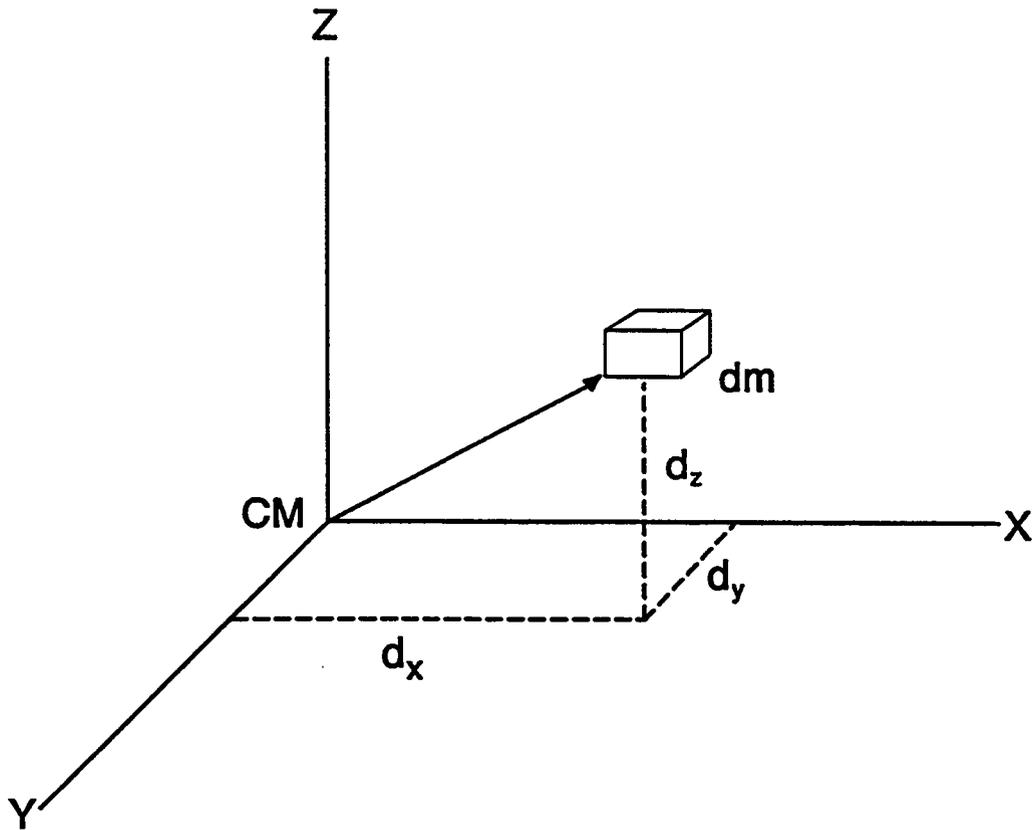


Figure 4. Differential mass with respect to the CM.

To experimentally determine P_{xy} , P_{xz} , and P_{yz} with the MPI, perform the following steps for the x-y plane, the x-z plane, and the y-z plane.

- a. Record the MOIs (I_{xx} , I_{yy} , and I_{zz}) that were determined during the basic MOI testing.
- b. Rotate the test part to an arbitrary axis (remaining within the x-y, y-z, or x-z planes) θ° from the coordinate axis (Figure 5). In order to simplify the calculations, a 45° rotation is suggested.
- c. Run an MOI test with the test part at this orientation. The resulting MOI values (I_{xy} , I_{xz} , or I_{yz}) provide the MOI of the test part with respect to a new coordinate system with axes within the x-y, x-z, and y-z planes of the basic coordinate system.

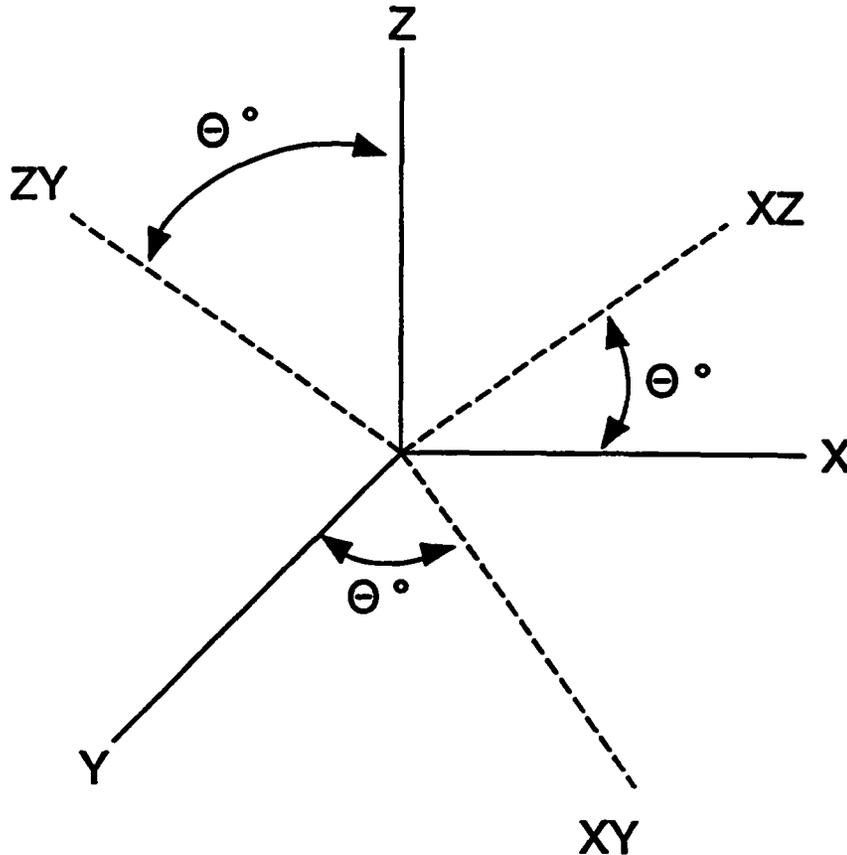


Figure 5. Arbitrary axis with respect to the coordinate axis.

d. Having calculated I_{xx} , I_{yy} , I_{zz} , I_{xy} , I_{xz} , and I_{yz} , the following transformation equations provide a means of solving for the products of inertia (Beer and Johnston, 396-397).

$$I'_{xy} = \frac{I_{xx} + I_{yy}}{2} + \frac{I_{xx} - I_{yy}}{2} \cos(2\theta) - P_{xy} \sin(2\theta)$$

$$I'_{xz} = \frac{I_{xx} + I_{zz}}{2} + \frac{I_{xx} - I_{zz}}{2} \cos(2\theta) - P_{xz} \sin(2\theta)$$

$$I'_{yz} = \frac{I_{yy} + I_{zz}}{2} + \frac{I_{yy} - I_{zz}}{2} \cos(2\theta) - P_{yz} \sin(2\theta)$$

The calculations are simplified by using 45° as the rotation angle ($\cos(2\theta)=0$ and $\sin(2\theta)=1$). After rearranging terms and reducing, P_{xy} , P_{xz} , and P_{yz} are solved by substituting the appropriate values into the following equations:

$$P_{xy} = \frac{I_{xx} + I_{yy}}{2} - I_{xy}$$

$$P_{xz} = \frac{I_{xx} + I_{zz}}{2} - I_{xz}$$

$$P_{yz} = \frac{I_{yy} + I_{zz}}{2} - I_{yz}$$

Principal moments and axes of inertia

For every three-dimensional object, there is a maximum, intermediate, and minimum MOI about the part's CM. These three moments are known as the principal MOIs. The axes about which these moments act are called principal axes of inertia. These axes define a unique coordinate system about which the products of inertia are equal to zero.

The principal MOIs are calculated by substituting the basic MOI and products of inertia into the following determinant equation:

$$\begin{vmatrix} I_{xx}-I & -P_{xy} & -P_{xz} \\ -P_{yx} & I_{yy}-I & -P_{yz} \\ -P_{zx} & -P_{zy} & I_{zz}-I \end{vmatrix} = 0.$$

The solution to this determinant is a cubic equation, whose roots yield three values for I. These values, I_1 , I_2 , and I_3 , yield the maximum, intermediate, and minimum values, respectively, of the test part's MOI.

To determine the orientation of the principal axes, the direction cosines (a, b, and c) for each axis must be determined. Direction cosines are the cosines of the angles between the principal axes (1, 2, and 3) and the basic reference axes (x, y, and z) (Figure 6). The direction cosines are calculated by substituting the basic MOIs, the products of inertia, and the individual principal MOIs (I_1 , I_2 , and I_3) into the equations which follow. This substitution yields three equations with three unknowns. The calculations must be performed three times, once for each of the principal moments of inertia.

$$(I_{xx} - I)a - P_{xy}b - P_{xz}c = 0$$

$$-P_{yx}a + (I_{yy} - I)b - P_{yz}c = 0$$

$$-P_{zx}a - P_{zy}b + (I_{zz} - I)c = 0.$$

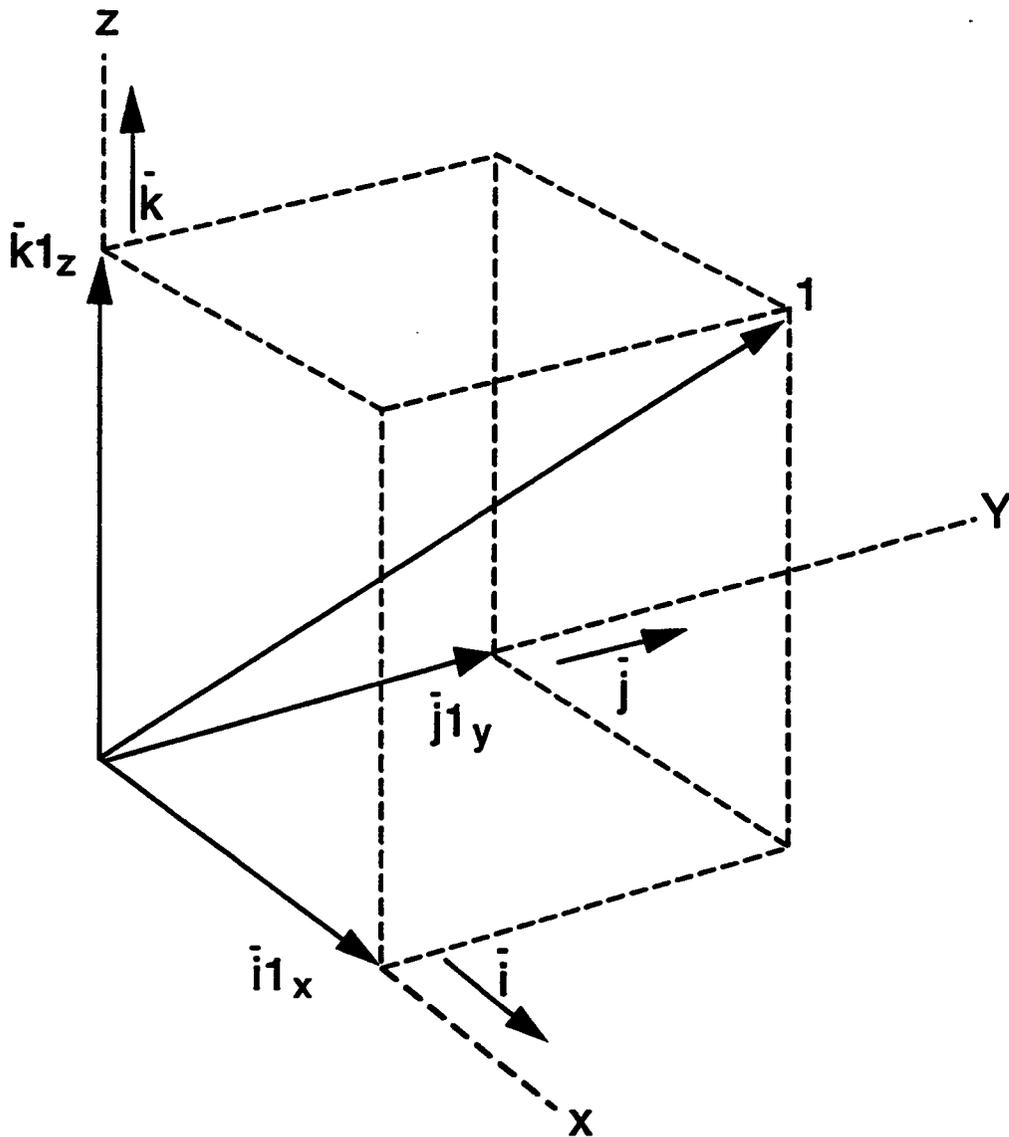


Figure 6. Direction cosines for principal axis 1.

The three calculations result in three sets of direction cosines:

- (1) $a_1, b_1, c_1,$
- (2) $a_2, b_2, c_2,$
- (3) $a_3, b_3, c_3.$

As a check, substitute each set of the direction cosines into the following equation:

$$a^2 + b^2 + c^2 = 1.$$

Each set of direction cosines defines a principal axis (1, 2, or 3) with respect to the basic coordinate axes. The relation between the principal axes and the coordinate axes is shown below in Figure 7 (Meriam and Kraige, 589-606).

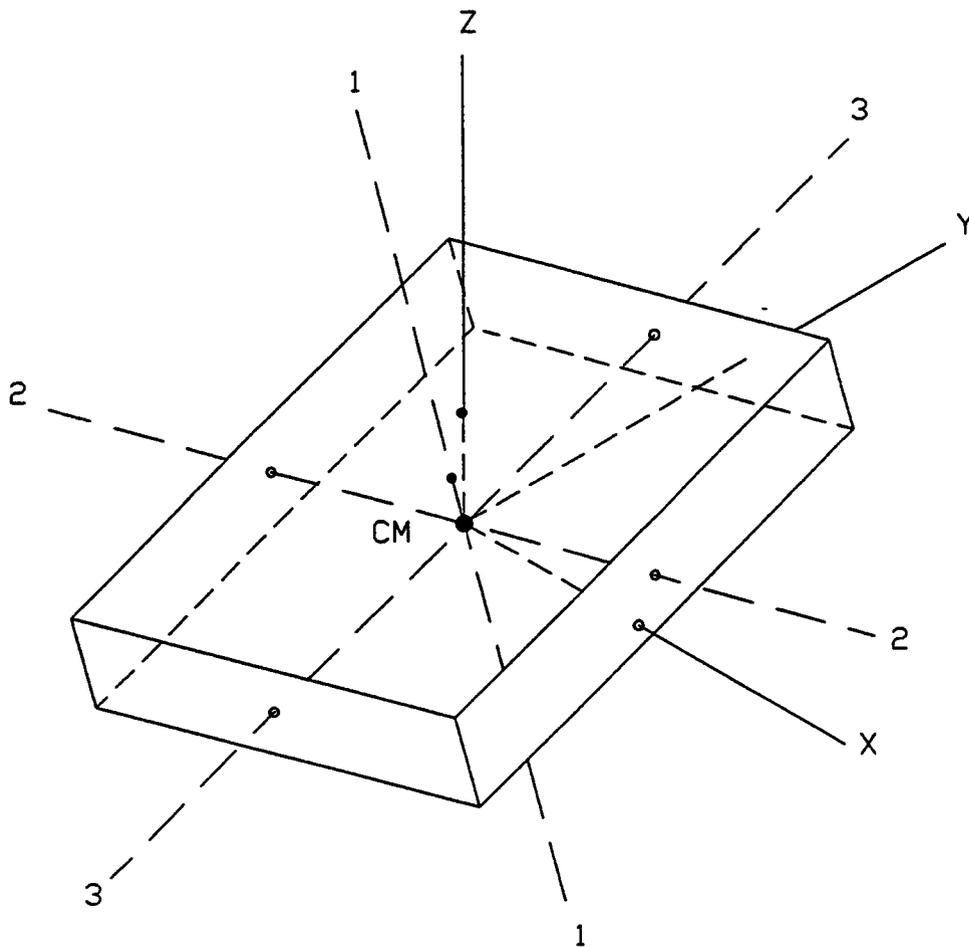


Figure 7. Relation between coordinate and principal axes.

Procedure of operation

The procedures to measure the CM and MOI of test parts are relatively simple, but must be carefully followed in order to ensure accuracy. Before operating the MPI, check for correct setup of the entire system as detailed in the MPI Model KSR330-60

instruction manual. The initial start up of the system is as follows:

1. Remove all objects from the MPI test platform.
2. Turn on the MPI, the gas bearing pressure, the air compressor, and the computer system. Allow at least 10 minutes for the console to warm up before operation of the MPI.
3. Execute the KSR program. If the program is not initiated, the rotary gas bearing system will not be supplied air by the compressor.
4. Press any key to display the main menu (Appendix C).
5. Select the update test information menu and enter the appropriate information. It is important to update the test part ID before each test performed.
6. Set the gas bearing pressure for the test part. To do so:
 - a. With the air off, mount the test fixture and the test part to the test platform.
 - b. Turn the air on and turn the pressure regulator counterclockwise until the bearing pressure gauge reads zero.
 - c. Slowly turn the regulator clockwise until the test platform and the test part float freely on the gas bearing.
 - d. Note the bearing pressure level.
 - e. Turn the regulator clockwise to get an additional 5 psi on the bearing pressure gauge. **WARNING:** Do not exceed 95 psi on the bearing pressure gauge.
 - f. Turn the air off at the toggle switch to lock the test platform in place and remove the test part from the fixture in order to prepare for the test sequence.

Calibration

Individual calibrations must be performed for the CM and the MOI. Both require the use of a calibration beam and weights (Figure 8). The calibration beam must be installed with holes A, B, and C in the quadrant between the +X and +Y axes. The computer will prompt the operator to place specific weights in particular holes on the calibration beam (Figure 9). The MPI system calibration essentially is a comparison of the measured CM

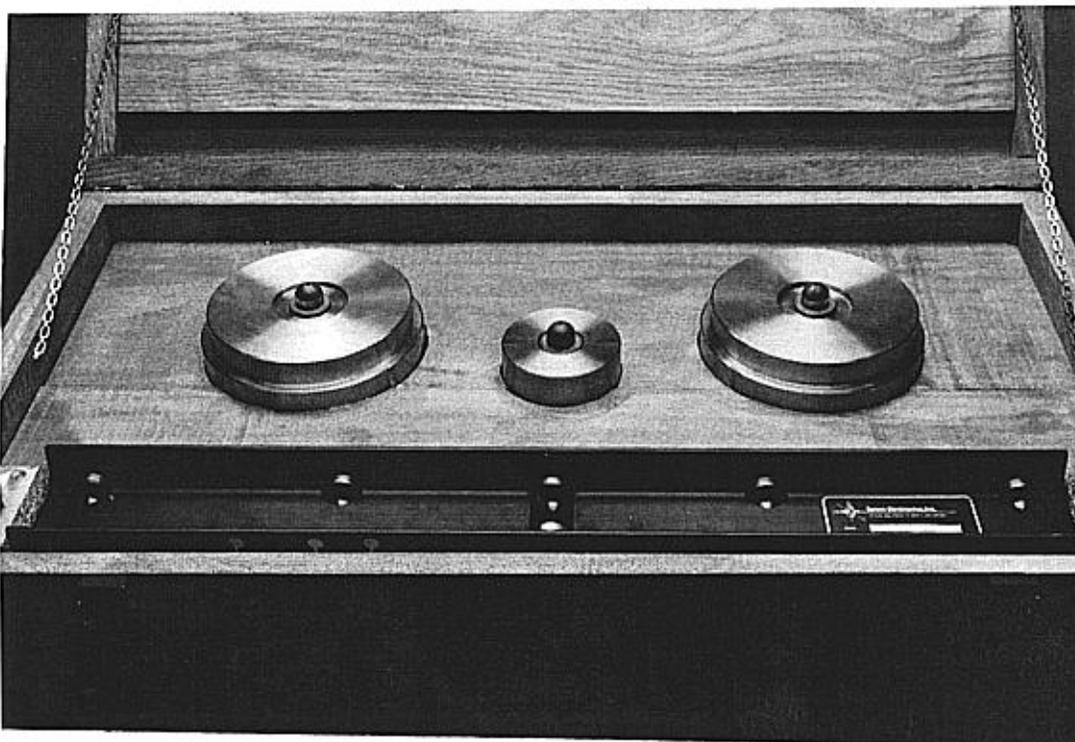


Figure 8. Calibration beam and weights.

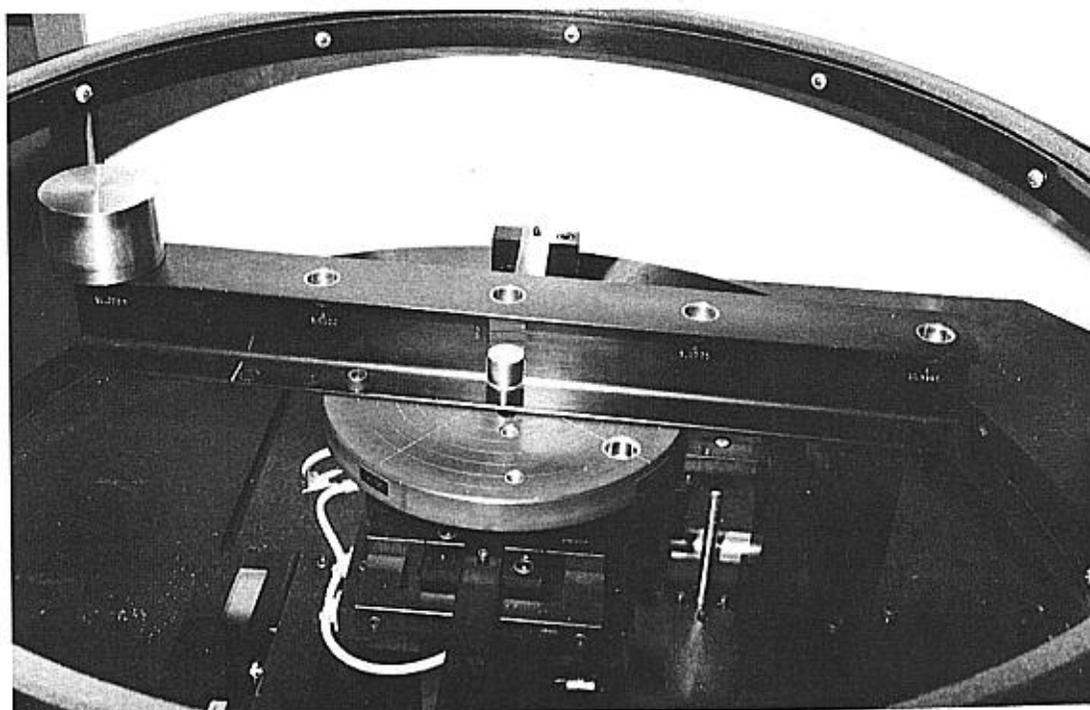


Figure 9. MPI setup for CM calibration.

and MOI values of the simple weight against their theoretical CM and MOI values. After the sequence is complete, the computer will display that the calibration was accepted or rejected. An accepted calibration indicates the machine is operating correctly and will provide reliable data. If rejected, the system is faulty and a diagnosis must be performed to find the source of error. Possible sources include: floor vibration, air drafts, touching the MPI during testing, improper system setup, etc.

Measurements

In order to measure the CM and MOI of a test part, the weight of the part and the estimated height of the part's CM above the test platform must be entered into the computer. The weight of the test part is crucial in obtaining accurate results and should be checked before each calculation is made. The estimated CM height has only a minor affect on the actual calculations of the CM. The measurements consist of three steps:

1. Tare measurement.
2. Part measurement.
3. Calculations.

Step 1: Tare measurement

The tare measurement is performed to eliminate the test fixture from the test part's CM and MOI calculations. During this measurement, the fixture to be used to mount the test part onto the platform should be attached. Test fixtures should be fabricated to mount to the test platform (Appendix D). It is important to mount the test fixture so that the defined test part coordinate system aligns with the test platform system. If an offset is present, it should be noted and be accounted for in the CM and MOI test results.

As with the calibration, individual tare measurements must be performed for the CM and the MOI. With the fixture in place, press the moment display button off and then on to zero the display. The operator then should follow the screen prompts to initiate the tare measurement. The computer will take readings from each quadrant and print the results. All subsequent part measurements will compensate for the test fixture's properties. If the fixture is to be placed in a different configuration, another tare measurement must be performed for the new fixture orientation.

Step 2: Part measurement

The test part now should be installed in the fixture and part measurement selected from the main menu. Again, the CM and MOI tests are two separate functions. For each test, the computer will prompt the operator through the sequence and print the raw data.

Step 3: Calculations

To obtain the test results, the calculations menu should be selected. The program provides the option to calculate the CM and MOI separately or together. After selecting which calculations are to be made, be sure to verify that the actual weight of the test part and the estimated CM height are entered properly. These values are crucial to obtaining correct data. The computer will perform the calculations and print the data. The MOI is given about the test part's CM and the platform's coordinate system.

Conclusions

After following the procedure to measure the mass properties of a test part, several conclusions were made:

1. The positioning of the fixture is crucial to obtain accurate results. It is necessary for the test part to be referenced to the pivot axis of the test platform. If the defined axis of the test part is not aligned with the pivot axis of the test platform, an offset distance will have to be figured into the results from the CM calculations.

2. Vertical components of the test fixture must be perpendicular to the test platform. The MPI is incapable of factoring the lean of the vertical components into the calculations. Therefore, the results will be erroneous if the fixture is not perpendicular.

3. It is important to update the test information before each test. By changing the test part ID each time, the results will be labeled appropriately on the printouts. This is very important when performing a series of tests.

4. Each time calculations are done, the mass of the test part should be entered. This will minimize the possibility of having the calculations based on the wrong test part weight.

5. The MPI must be absolutely isolated from any vibration in the surrounding environment. Vibration isolators were added to the described MPI system.

6. A test fixture may require the part to be attached in such a way that would cause an offset between the center axis of the test part and that of the test platform. If this is the case, be sure to attach the part the same way each time so that the offset is always in the same direction.

7. The maximum moment allowed due to the offset of the CM of a test part is 60 lb/in. When designing test fixtures, this restriction must be kept in mind.

8. Accurate results are obtained by running both the CM and MOI tests together and then performing the calculations for both. The MPI calculates the MOI results about the test part's CM. If the CM was never determined, the MOI results will be based on the CM of the last part tested and will be wrong.

References

- Beer, Ferdinand P., and Johnston, Jr., E. Russell. 1977. Vector mechanics for engineers: Statics and dynamics. 3rd ed. New York: McGraw-Hill Book Company.
- Meriam, J. L., and Kraige, L. G. 1986. Engineering mechanics: Dynamics. Vol 2. 2nd ed. New York: John Wiley and Sons.
- Space Electronics, Inc. n.d. Mass Properties Instrument Model KSR330-60 instruction manual. Berlin, CT: Space Electronics, Inc.

Appendix A.

Accuracy of the MPI

The accuracies of the MPI are as follows:

CM error for 100-lb part with 0.5-in. CM offset

Bearing/machine tolerance.	0.001000 in.
Sensitivity error (0.003/part weight in lbs.	0.000030 in.
Linearity error (0.03% of full scale).	0.000180 in.
Max uncertainty this example.	<u>0.001210 in.</u>

Moment of inertia

Basic accuracy	$\pm 0.25\%$
Tare MOI (typical)	130 lb-in ²
MOI error for 800 lb-in ² part:	
Basic accuracy	2.0 lb-in ²

Appendix B.

Determination of torsional pendulum physical constant

The calibration constant primarily is a function of the spring rate of the torsion rod. A calibration weight is placed at a known distance from the center of the test platform and the period of oscillation (T_c) is measured. The weight then is placed at the center of the platform and a second period (T_o) is found. The difference between the two readings is completely due to the change in MOI. The MOI change, I , is defined as $I = WR^2$, where W is the calibration weight and R is the offset. The calibration constant then is determined:

$$C = I / (T_c^2 - T_o^2).$$

Appendix C.

KSR main menu

The main menu of the KSR program appears as follows:

>>>> MAIN MENU <<<<

F1: Update test information
F2: Part measurement
F3: Tare measurement
F4: Calibration
F5: System utilities
F6: Calculations
<ESC>: Quit

Select function from list

Appendix D.

Mounting of test fixture

Any part to be tested must be interfaced with the test platform by means of a test fixture. The fixture must be fabricated to fit the hole pattern of the test platform. This pattern is detailed in Figure D-1 shown below.

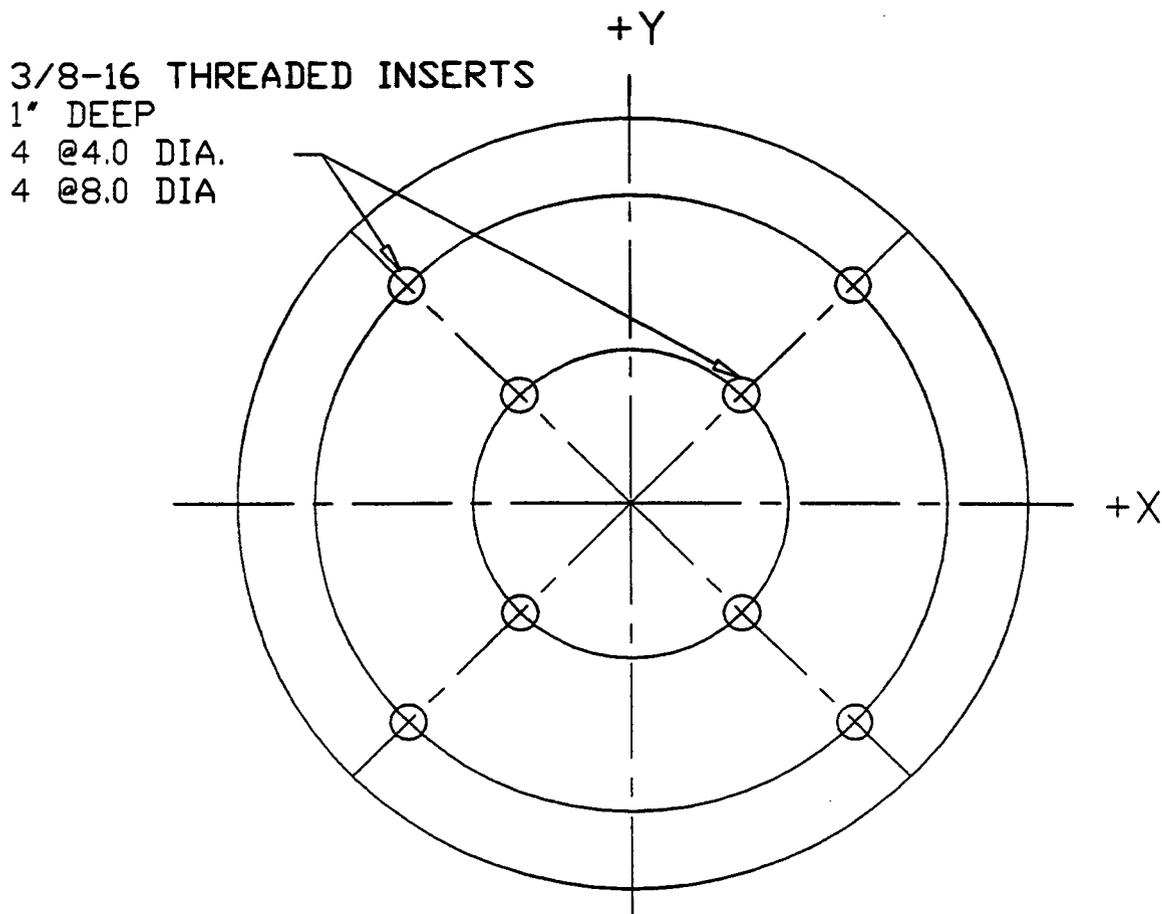


Figure D-1. Mounting pattern of test platform.

Appendix E.

Manufacturers' list

Space Electronics, Inc.
81 Fuller Way
Berlin, CT 06037

JUN-AIR (USA) Inc.
1303 Barclay Blvd.
Buffalo Grove, IL 60089