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THE USE OF OPAQUE LOUVRES AND SHIELDS TO REDUCE REFLECTIONS WITHIN THE COCKPIT: A MATHEMATICAL TREATMENT

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SUMMARY

Opaque shields can be used to channel light and thereby reduce reflections in the cockpit. These shielding devices range from the standard glare shield on top of the instrument panel to the more experimental use of Light Control Film[R] and Micromesh[R] for this purpose. Because of the need to determine the best position, width, spacing, etc. of these shielding devices, it was felt that a systematic approach would be highly desirable. This work shows a mathematical approach to this problem and includes derivations, examples, and a suggested figure of merit.

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Commanding

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INTRODUCTION

One technique of reducing the reflections of the instruments, dials, etc. from the transparent enclosures is the use of opaque louvers and shields. In using these screening materials, one wants to maximize the extent to which they block light from reaching the canopy but minimize the extent to which they block light from reaching the pilots' eyes. This is accomplished by choosing the proper values for the position, width, spacing, angle, etc. of these shields. The present work was inspired by the need to determine the proper thickness, i.e., louvre width, of the Light Control Film[R] which we are planning to examine as a potential means of reducing reflections within the cockpit. This film, a product of the 3M Co., consists of a thin sheet of plastic incorporating many thin louvres, .0003 inch thick and oriented normally to the surface. These sheets can be obtained in varying thicknesses from .015 to .030 and can be placed directly on the instrument or other source of unwanted reflections. Although this analysis was performed in connection with a specific material whose width is the only parameter that can be varied, it was made to apply to the general case in order to extend its usefulness. For example, this analysis could be used in connection with selecting the proper parameters for the standard instrument panel glare shield.

In this study, we use the method of analytical geometry to solve the complicated visibility problem. In order to simplify the matter, only a two-dimensional case is considered. The Cartesian coordinates are used and the location of each point in 2-D space may be represented by a set of the coordinate numbers (x,y). The pilot's eye is at point P in Figure 1. His height, in a sitting position, from his eye to the aircraft floor is denoted as Py. The axis of point P to the ground is chosen to coincide with the y-axis and the horizontal level of the aircraft is the x-axis. "a" is the distance from the origin (o,o) to the intersection of the floor with the extension of the plane of the instrument panel at point A(a,o). The shield whose base point is at B(bx,by) and which is b meters from point A has a width d_2. The shield whose base point is at C(cx,cy) and which is b+c meters from point A has a width d_1. The distance between these two shields is c. The corresponding Cartesian coordinates of all specified points in Figure 1 are attached in Appendix I.

Points H and G are the y-axis extension points of B-F and C-F, respectively. Θ is the decline angle of the panel. P can be: (I) above H (i.e. Py > Hy), (II) between H - G (i.e. Hy > Py > Gy) and

(III) below G (i.e. $Gy > Py$). Thus the problem can be subdivided into three different cases. P^1 is the projection point from P to E (or F) onto the AD line. We define the visibility as the percent of BP^1 over BC (in Case I) and CP^1 over BC (in Case III). Since Case II is a trival case, only Cases I and III are mathematically derived.

SOLUTION

The detailed derivation of Cases I and III are attached in Appendices II and III respectively. The results are shown as follows:

(I) $V_I = 1 - \dfrac{d_1}{c} \dfrac{\tan \Theta - k_1}{1 + k_1 \tan \Theta}$ $\qquad Py > Hy$

(II) $V_{II} = 1.0$ (or 100%) $\qquad Hy > Py > Gy$

(III) $V_{III} = 1 - \dfrac{d_2}{c} \dfrac{\text{Cot } \Theta - k_2}{1 + k_2 \text{ Cot } \Theta}$ $\qquad Gy > Py$

where $k_1 = \dfrac{a + (b+c) \text{ Cos } \Theta - d_1 \sin \Theta}{Py - [(b+c) \text{ Sin } \Theta + d_1 \text{ Cos } \Theta]}$

and $k_2 = \dfrac{a + b \text{ Cos } \Theta - d_2 \text{ Sin } \Theta}{Py - b \text{ Sin } \Theta - d_2 \text{ Cos } \Theta}$

Let us assume $d_1 = d_2 = 0.1\text{m}$, $\Theta = 60°$, $a = 4\text{m}$, $b = 2\text{m}$ and $c = 1\text{m}$. Then the numerical relation between V and Py can be expressed as follows. (Derivation is attached in Appendix IV.)

$$V_I = \frac{0.827\ Py + 5.89}{Py + 4.99} \qquad \text{for } Py > Hy\ (= 5.77\text{m})$$

$$V_{II} = 1.0\ (\text{ or } 100\%) \qquad Hy > Py > Gy\ (= 4.62\text{m})$$

$$V_{III} = \frac{0.943\ Py + 0.42}{Py + 1.01} \qquad Gy > Py$$

A plot of this relationship is given in Figure 3 (tabulated in Table 1). In this example, Hy, Gy, can be formulated as shown in Appendix I. From Figure 3, the curve is noted to be highly nonlinear and non-symmetric. A family of reference curves can be easily generated from the computer. Since this report concentrates on the mathematical analysis, actual computation to a specific aircraft will be done elsewhere. Nevertheless, this report computes the case where $\Theta = 90°$, and $\Theta = 60°$. The rest of Θ can be extrapolated from these curves. Results are in Figure 4. Their corresponding numberical values are shown in Appendices V and VI and in Tables 2 and 3.

Actually, of course, we are interested in maximizing V over some Py_s to Py_t of probable eye positions rather than at a single point, i.e., we want to maximize the integral of V over the domain Py_s-Py_t. Also we want to minimize the value of V at the point where the y-axis intersects the canopy. Therefore, an appropriate figure of merit might be a formula such as

$$\left(\int_{Py_s}^{Py_t} V\, dPy \right) - zV_1$$

where V_1 is the value of V at the point where the y-axis intersects the canopy and z is an empirical constant, based upon human factors data, used to establish the relative weights between the two terms.

In conclusion, we have used a mathematical analysis to derive a general analytic equation for aircraft cockpit visibility. An example of the computation is given and the value of the area over the range of interest is suggested to be a simple referential value for determining the pilot's visibility.

TABLE 1. NUMERICAL VALUE OF VISIBILITY VERSUS Py

Assume $d_1 = d_2 = 0.1m$ $\theta = 60°$ $a = 4m$

$b = 2m$ $c = 1m$

Case I Py > Hy, formula $VI = \frac{0.827\ Py + 5.89}{Py + 4.99}$

Case III Py < Gy, formula $V_{III} = \frac{0.943\ Py + 0.42}{Py + 1.01}$

PY	V_I	V_{II}	V_{III}
19.16	.90		
.	.		
.	.		
.	.		
.	.		
.	.		
.	.		
7.0	.97		
6.5	.97		
6.0	.99		
5.77			
4.62		1.00	
4.0			.837
3.0			.81

TABLE 2. NUMERICAL VALUE FOR APPENDIX V

$\theta = 90°$ $d_1 = d_2 = 0.01$ m $a = 4$ m $b = 2$ m $c = 0.1$ m

$Hy = 2.1$ $Gy = 2.0$

$$V_I = \frac{37.8 - Py}{39.9}$$

Py	V_I	V_{II}	V_{III}
5.9		0.800	
.		.	
.		.	
.		.	
2.4		0.887	
2.3		0.888	
2.2		0.890	
2.1		1.0	
2.0		1.0	
1.9			0.89
1.8			0.888
1.7			0.887
.			
.			
.			
.			
.			
.			
.			
0			0.845

TABLE 3

Example 2

$\theta = 60°$ $d_1 = d_2 = 0.01$ m $a = 4$ m $b = 2$ m $c = 0.1$ m

Hy = 4.73   Gy = 4.62

$$V_I = \frac{0.957\ Py - 0.225}{Py + 0.358} \qquad V_{III} = \frac{0.942\ Py - 1.74}{Py + 1.14}$$

Py	V_I	V_{II}	V_{III}
.			
.			
.			
.			
5.33	.83		
5.13	.84		
4.93	.85		
4.73		1.00	
4.62		1.00	
4.42			0.44
4.22			0.41
4.02			0.39

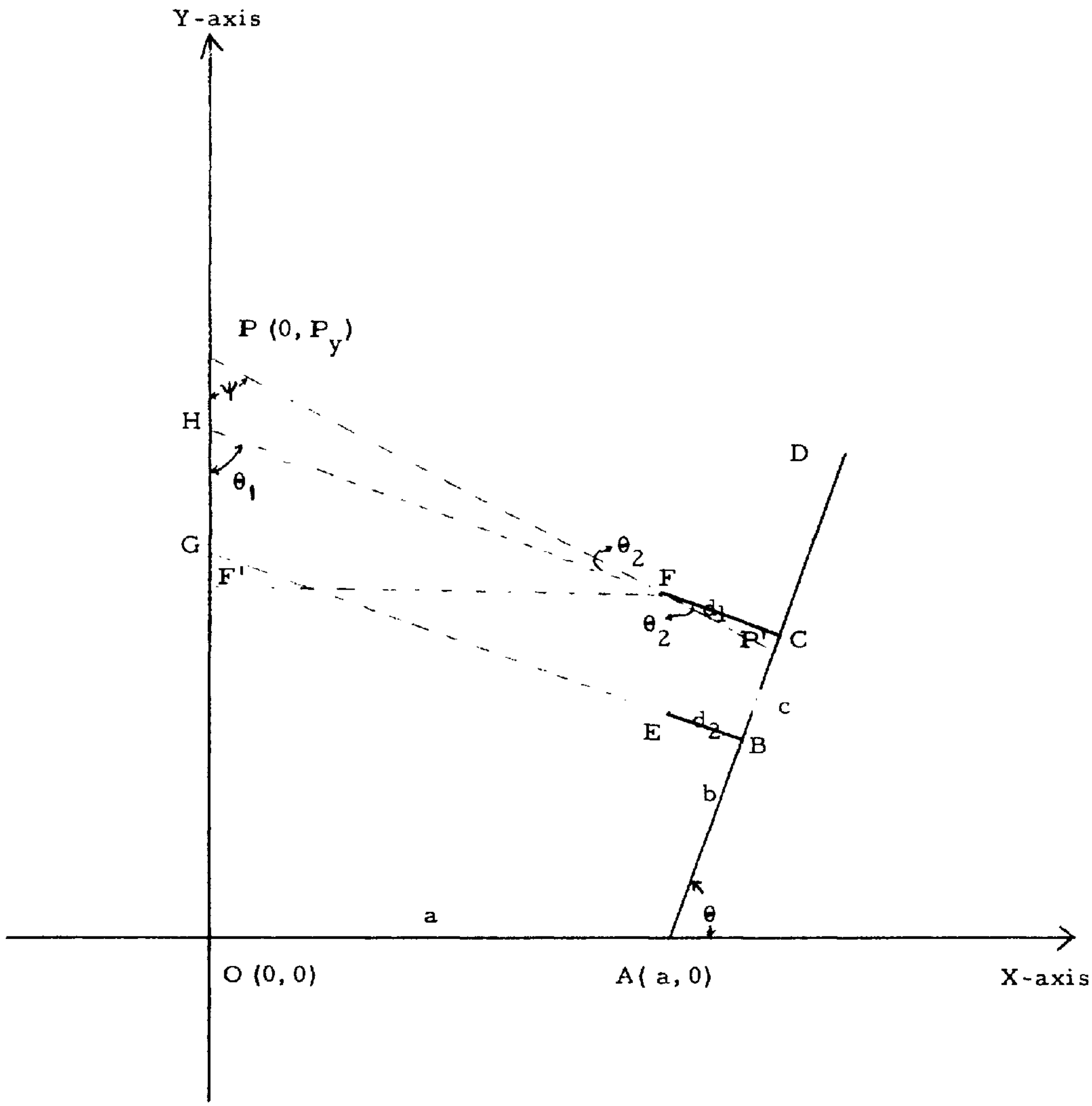

Figure 1 Schematic for case I.

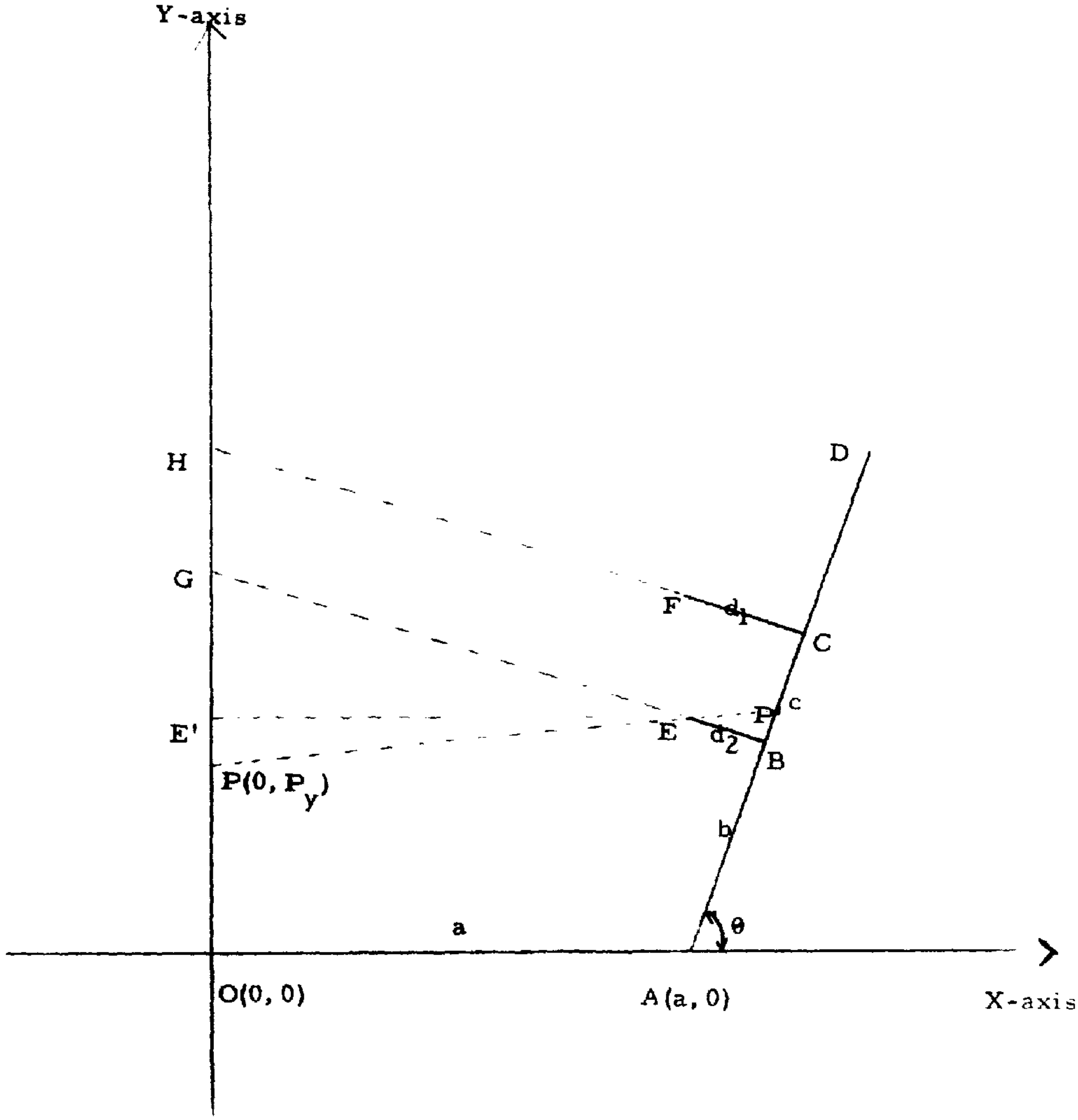

Figure 2   Schematic for case III.

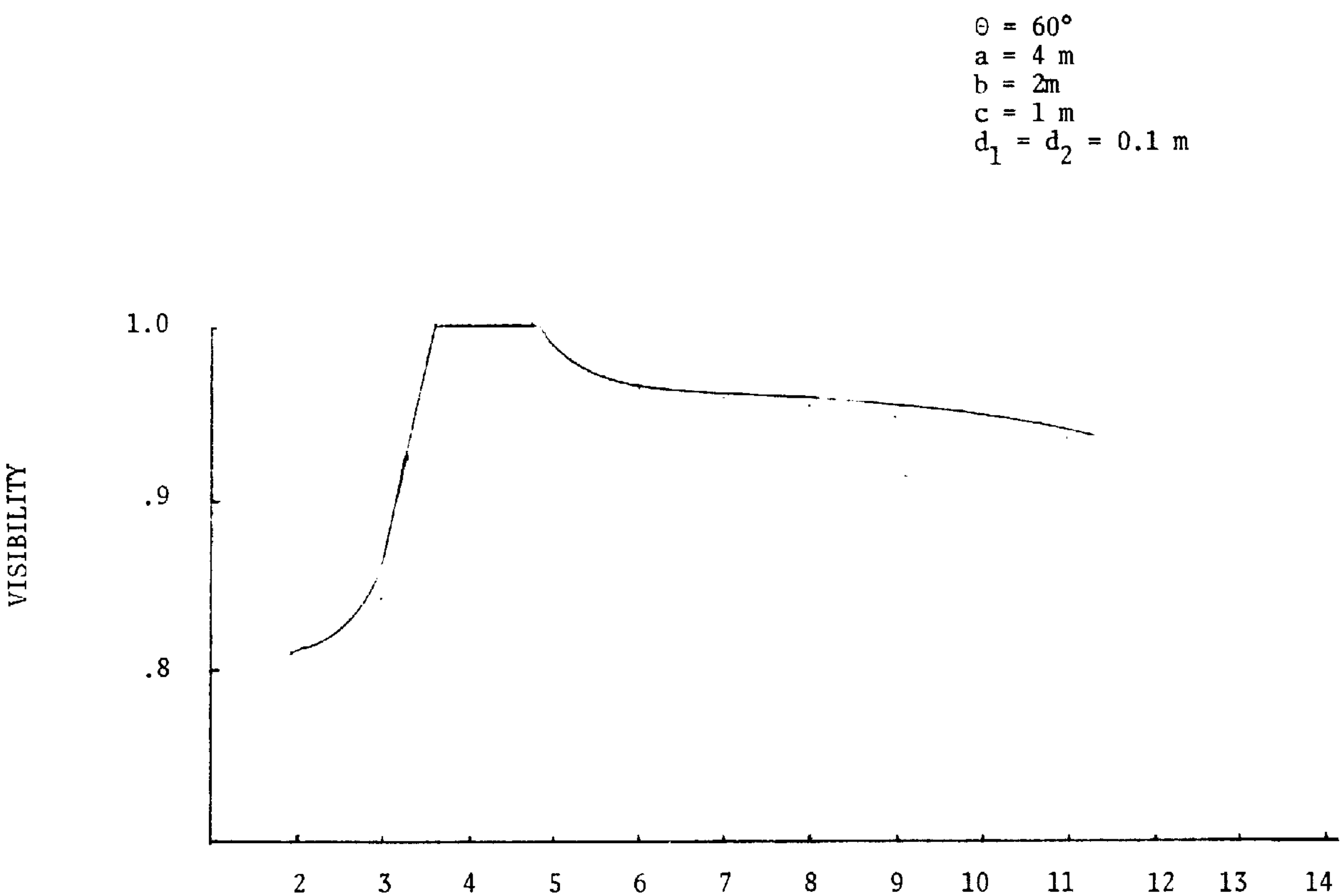

FIGURE 3. Plot of Py vs. Visibility

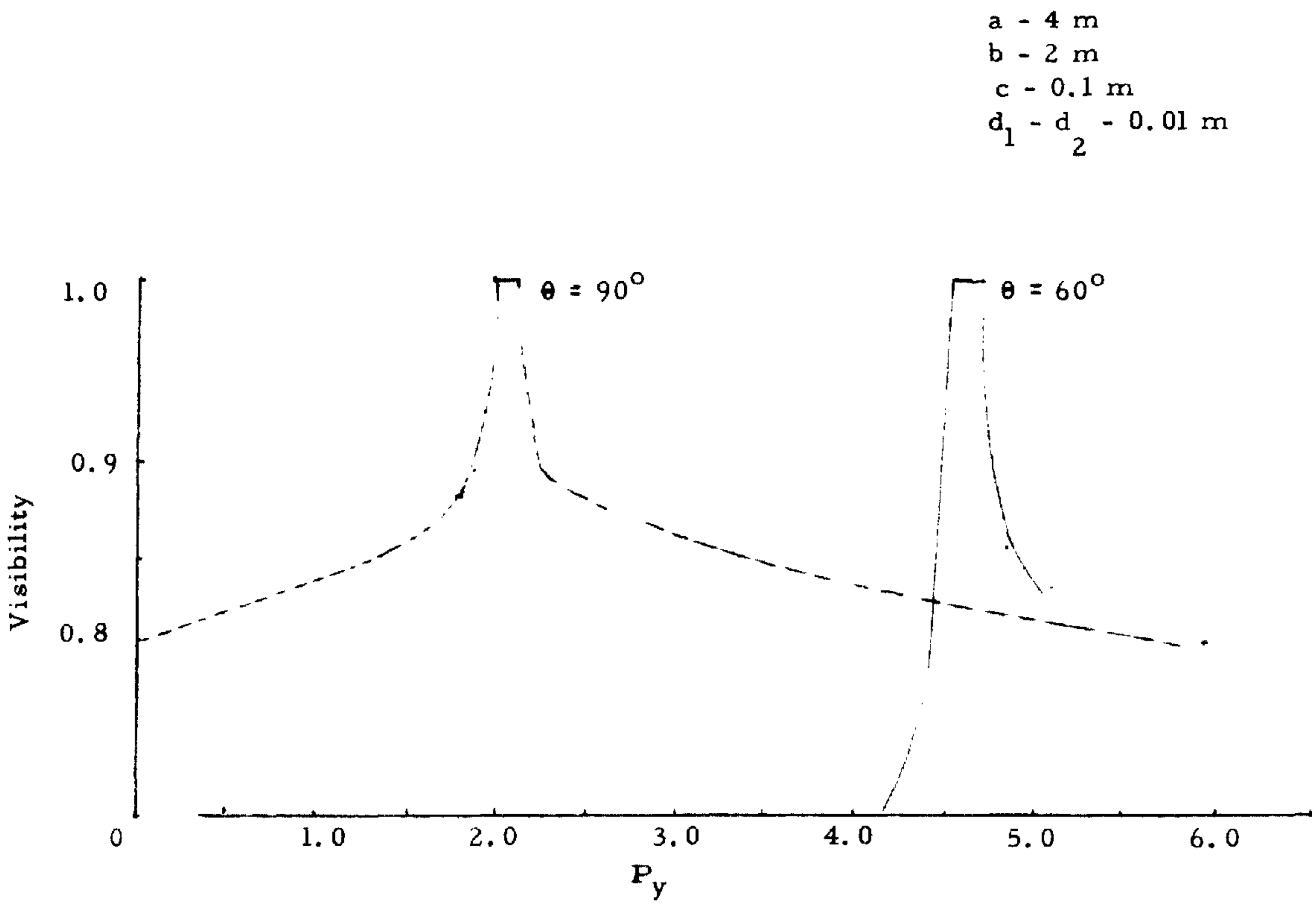

FIGURE 4. Plot of P_y vs. Visibility

APPENDIX I. CARTESIAN COORDINATES OF POINTS

$Ax = a$	$Ay = 0$	$A\ [a, o)$
$Bx = a + b \cos \theta$	$By = b \text{ Sin } \theta$	$B\ [a+b \cos \theta,\ b \sin \theta]$
$Cx = a + (b+c) \cos \theta$	$Cy = (b+c) \sin \theta$	$C\ [a+(b+c)\cos \theta,\ (b+c) \sin \theta]$
$Ex = [(a+b \cos \theta) - d_2 \sin \theta]$	$Ey = b \text{ Sin } \theta + d_2 \cos \theta$	$E\ [(a+b \cos \Theta) - d_2 \text{ Sin } \Theta,\ b \text{ Sin } \Theta + d_2 \text{ Cos } \Theta]$
$Fx = [(a+(b+c) \cos \theta) - d_1 \text{ Sin } \theta]$	$Fy = (b+c) \text{ Sin } \theta + d_1 \text{ Cos } \theta$	$F\ [(a+(b+c) \cos \theta) - d_1 \text{ Sin } \theta,\ (b+c) \text{ Sin } \Theta + d_1 \text{ Cos } \Theta]$
Py = variable		$P\ [0, Py]$
θ = variable		
For $0 < \theta < \frac{\Pi}{2}$		
$Hy = (a+(b+c) \text{ Sec } \theta) \tan (\frac{\Pi}{2} - \theta)$		$H\ [0, (a+(b+c) \text{ Sec } \theta) \tan (\frac{\Pi}{2} - \theta)]$
$Gy = (a+b \text{ Sec } \theta) \tan (\frac{\Pi}{2} - \theta)$		$G\ [0, (a+b \text{ Sec } \theta) \tan (\frac{\Pi}{2} - \theta]$
For $\theta = \frac{\Pi}{2}$		
$Hy = b + c$		$H\ [o, b+c)$
$Gy = b$		$G\ (o, b)$

APPENDIX II. DERIVATION OF CASE I

Let us define ψ, Θ_1 and Θ_2 as in Figure 1. Let F^1 be the point at the y-axis of F which is parallel to the x-axis. Then by the simple trigonometrical relationship, it follows for Case I.

(Case I)

$$\psi = \tan^{-1} \frac{F F^1}{PO - F^1 0} = \tan^{-1} \frac{Fx}{Py - Fx}$$

Since Fx is given in Appendix I. Thus

$$\psi = \tan^{-1} \frac{(a+(b+c) \text{ Cos } \Theta - d_1 \sin \Theta)}{Py - [(b+c) \sin \Theta + d_1 \cos \Theta]}$$

Futhermore,

$$\Theta = \psi + \Theta_2$$

Thus

$$\Theta_2 = \Theta - \psi = \Theta - \tan^{-1} \frac{(a + (b+c) \cos \Theta - d_1 \sin \Theta)}{Py - [(b+c) \sin \Theta + d_1 \cos \Theta]}$$

then

$$P^1C = d_1 \tan \Theta_2 = d_1 \tan \left(\Theta - \tan^{-1} \frac{[a+(b+c) \cos \Theta - d_1 \sin \Theta]}{Py - [(b+c) \text{ Sin } \Theta + d_1 \cos \Theta]} \right)$$

By the tangent law, it can be simplified as

$$P^1C = d_1 \left(\frac{\tan \Theta - k}{1 + k \tan \Theta} \right)$$

where

$$k = \frac{a + (b+c)\cos\theta - d_1 \sin\theta}{Py - [(b+c)\sin\theta + d_1 \cos\theta]}$$

Finally visibility V % is derived as,

$$V_I = \frac{P^1B}{BC} = 1 - \frac{P^1C}{BC} = \left[1 - \frac{d_1}{c}\left(\frac{\tan\theta - k}{1 + k\tan\theta}\right)\right]$$

APPENDIX III. DERIVATION OF CASE III

From Figure 2

$$pEy = py - Ey = [py - (b \sin\theta + d_2 \cos\theta)]$$

$$\theta_4 = \tan^{-1}\frac{PEx}{Ex} = \tan^{-1}\left(\frac{py - b\sin\theta - d_2\cos\theta}{a+b\cos\theta - d_2 \mathrm{Sin}\ \theta}\right)$$

$$= \tan^{-1}[1/k_2] \text{ where } k_2 \text{ is the value of } 1/[\quad]$$

$$\theta_2 = \left(\frac{\Pi}{2} - \theta\right) - \theta_4 = \frac{\Pi}{2} - \theta - \tan^{-1}[1/k_2]$$

$$Bp^1 = d_2 \tan\theta_2 = d_2 \tan\left(\frac{\Pi}{2} - (\theta + \tan^{-1}(1/k_2))\right)$$

$$= d_2 \mathrm{Cot}\ (\theta + \tan^{-1}(1/k_2)) = d_2\ \frac{\mathrm{Cot}\,\theta - \cot\tan^{-1}(1/k_2)}{1 + \mathrm{Cot}\,\theta\,\mathrm{Cot}\tan^{-1}(1/k^2)}$$

$$\doteq d_2\ \frac{\mathrm{Cot}\,\theta - k_2}{1 + k_2 \cot\theta}$$

Thus

$$V_{III} = 1 - \frac{BP^1}{C} = 1 - \frac{d_2}{C}\left(\frac{\mathrm{Cot}\,\theta - k_2}{1 + k_2\,\mathrm{Cot}\ \theta}\right)$$

APPENDIX IV

Example 1.

For simplicity, let $d_1 = d_2 = 0.1\text{m}$, $\theta = 60°$, $a = 2b = 4c = 4\text{m}$ (i.e. $a = 4\text{m}$, $b = 2\text{m}$, $c = 1\text{m}$)

$$k_1 = \frac{4 + (2 + 1)\ \text{Cos}\ 60° - 0.1\ \text{Sin}\ 60°}{Py - (2+1)\ \sin 60° + 0.1\ \text{Cos}\ 60°}$$

$$= \frac{4 + 3 \times 0.5 - 0.1 \times 0.87}{Py - (3 \times 0.87 + 0.1 \times 0.5)}$$

$$= \frac{4.413}{Py - 2.65}$$

and

$$k_2 = \frac{4 + 2 \cos 60° - 0.1\ \text{Sin}\ 60°}{Py - 2 \sin 60° - 0.1 \cos 60°}$$

$$= \frac{4 + 2 \times 0.5 - 0.1 \times 0.87}{Py - 2 \times 0.87 - 0.1 \times 0.5}$$

$$= \frac{4.913}{Py - 1.79}$$

Thus

$$V_I = 1 - \frac{0.1}{1} \quad \frac{\tan 60° - \dfrac{4.413}{Py - 2.65}}{1 + \dfrac{4.413}{Py-2.65} \tan 60°}$$

$$= 1 - 0.1 \; \frac{1.73\ (Py - 2.65)\ -4.413}{(Py - 2.65) + 4.413 \times 1.73}$$

$$= 1 - \frac{0.173\ Py - 0.90}{Py + 4.99} = \frac{.827\ Py + 5.89}{Py + 4.99}$$

$$V_{III} = 1 - \frac{0.1}{1} \; \frac{\text{Cot } 60° - \frac{4.913}{Py - 1.79}}{1 + \frac{4.913}{Py - 1.79}\ \text{Cot } 60°}$$

$$= 1 - 0.1 \; \frac{.57\ Py - .57 \times 1.79 - 4.913}{Py - 1.79 + 4.913 \times .57}$$

$$= 1 - \frac{.057\ Py - 0.594}{Py + 1.01}$$

$$= \frac{0.943\ Py + .42}{Py + 1.01}$$

Thus

$$V_I = \frac{0.827\ Py + 5.89}{Py + 4.99}$$

$$V_{II} = 1.0 \text{ (or 100\%)}$$

$$V_{III} = \frac{0.943\ Py + 0.42}{Py + 1.01}$$

And

$$Hy = (a+(b+c)\ \mathrm{Sec}\ \theta)\ \tan\left(\frac{\Pi}{2} - \theta\right) = (4 + (2+1)\ \mathrm{Sec}\ 60°)\ \tan\left(\frac{\Pi}{2} - 60°\right)$$

$$= (4 + 3 \times 2) \times 0.577 = 5.77\ \mathrm{m}$$

$$Gy = (a+b\ \sec\theta)\ \tan\left(\frac{\Pi}{2} - \theta\right) = (4 + 2\ \sec\ 60°)\ \tan\left(\frac{\Pi}{2} - 60°\right)$$

$$= (4 + 4) \times 0.577 = 4.62\ \mathrm{m}$$

APPENDIX V

Example 2

Let $\theta = 90°$, $d_1 = d_2 = 0.01m$ $a=4m$ $b = 2m$ $c = 0.1m$

by (1)

$$k_1 = \frac{4+(2+0.1)\ Cos\ 90° = 0.01\ Sin\ 90°}{Py - [(2+0.1)\ Sin\ 90° + 0.01\ Cos\ 90°]} = \frac{3.99}{Py - 2.1}$$

$$V_I = 1 - \frac{0.01}{0.1} \frac{\tan 90° - \frac{3.99}{Py - 2.1}}{1 + \frac{3.99}{Py - 2.1} \tan 90°} = 1 - (0.1) \frac{Py - 2.1}{3.99}$$

$$= \frac{39.9 - Py - 2.1}{39.9} = \frac{37.8 - Py}{39.9}$$

by (III)

$$k_2 = \frac{4 + 2\ Cos\ 90° - 0.01\ Sin\ 90°}{Py - 2\ Sin\ 90° - 0.01\ Cos\ 90°} = \frac{3.99}{Py - 2}$$

$$V_{III} = 1 - \frac{0.01}{0.1} \frac{Cot\ 90 - \frac{3.99}{Py - 2}}{1 + \frac{3.99}{Py-2}\ Cot\ 90} = 1 + 0.1 \times \frac{3.99}{Py-2}$$

$$= \frac{Py - 2 + 3.99}{Py - 2} = \frac{Py - 1.601}{Py - 2}$$

Note since $\epsilon = 90°$ the case shall be symmetric for V_I, V_{III}. We need to compute case V_I only.

Compute Hy, Gy

$$Hy = (b+c) = 2 + 0.1 = 2.1$$

$$Gy = b = 2.0$$

APPFNDIX VI

Example 3

Let $\theta = 60°$ $d_1 = d_2 = 0.01m$ $a = 4m$ $b = 2m$ $c = 0.1m$

by (I)

$$k_1 = \frac{4 + (2 + 0.1) \text{ Cos } 60° - 0.01 \text{ Sin } 60°}{Py - [(2 + 0.1) \text{ Sin } 60° + 0.01 \text{ Cos } 60°]}$$

$$= \frac{4 + (2.1)\ (a5) - (0.01)\ (0.866)}{Py - [(2.1)\ (0.866) + (0.01)\ (0.5)]}$$

$$= \frac{4 + 1.05 - 0.00866}{Py - [1.8186 + .005]} = \frac{5.04}{Py - 1.824}$$

$$V_I = 1 - (0.1) \frac{\tan 60° - \frac{5.04}{Py - 1.824}}{1 + \frac{5.04}{Py - 1.824} \tan 60°}$$

$$= 1 - (0.1) \frac{.433 - \frac{5.04}{Py - 1.824}}{1 + \frac{(5.04 x (.433))}{(Py - 1.824)}}$$

$$= 1 - (0.1) \frac{(0.433)\ Py - (0.433)\ (1.824) - 5.04}{(Py - 1.824) + (5.04)\ (0.433)}$$

$$= \frac{(Py + 0.358) - 0.0433\ Py - 0.583}{Py + 0.358}$$

$$= \frac{0.957\ Py + 0.225}{Py + 0.358}$$

$$k_2 = \frac{4 + 2 \text{ Cos } 60° - 0.01 \text{ Sin } 60°}{Py - 2 \text{ Sin } 60° - 0.01 \text{ Cos } 60°} = \frac{4 + 1 - 0.00866}{Py - 1.732 - 0.005}$$

$$= \frac{4.99134}{Py - 1.737}$$

$$V_{III} = 1 - (0.1) \frac{\text{Cot } 60° - \frac{4.99}{Py - 1.74}}{1 + \frac{4.99}{Py - 1.74} \quad \text{Cot } 60°}$$

$$= 1 - (0.1) \frac{0.577\ Py - 1.005 - 4.99}{Py - 1.74 + 2.88}$$

$$= \frac{Py - 1.14 - 0.058\ Py - 0.5995}{Py + 1.14}$$

$$= \frac{0.942\ Py - 1.74}{Py + 1.14}$$

$$Hy = (4 + 2.1 \text{ Sec } 60°) \tan (30°) = (4 + (2.1)\ (2)) \times 0.577$$

$$= 4.73$$

$$Gy = (4 + 2 \text{ Sec } 60°) \tan 30'' = (4 + 4) \times 0.57)$$

$$= 4.62$$